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CONTENTS

Editorial Board pages	iii
Call for Papers: IJRM Special Issue on Branding in a Digitally-Empowering World	v
Call for Papers: Marketing and Innovation	vi
J. Goldenberg and E. Muller, Editorial	99
Regular Papers	
G. Liberali, G.L. Urban and J.R. Hauser, Competitive information, trust, brand consideration and sales: Two field experiments	101
F. Germann, G.L. Lilien and A. Rangaswamy, Performance implications of deploying marketing analytics	114
W.A. Kamakura and C.W. Schimmel, Uncovering audience preferences for concert features from single-ticket sales with a factor-analytic random-coefficients model	129
S. Feld, H. Frenzen, M. Krafft, K. Peters and P.C. Verhoef, The effects of mailing design characteristics on direct mail campaign performance	143

(Contents continued on back cover)

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Uncovering audience preferences for concert features from single-ticket sales with a factor-analytic random-coefficients model

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ABSTRACT

To better plan their programs, producers of performing arts events require forecasting models that relate ticket sales to the multiple features of a program. The framework we develop, test and implement uncovers audience preferences for the features of an event program from single-ticket sales while accounting for interactions among program features and for preference heterogeneity across markets. We develop a factor-analytic random-coefficients model that overcomes four major methodological challenges. First, the historical data available from each market is limited, preventing the estimation of models at the market level and requiring some form of shrinkage estimator that also takes into account the diversity in preferences across markets as well as the fact that preferences for the many (26 in our application) program features are correlated across markets, requiring the estimation of a large covariance matrix for these preferences across markets. Our proposed factor-analytic regression formulation parsimoniously captures the principal components of the correlated preferences and provides shrinkage estimates at the individual market level. The second challenge we face is the fact that orchestras differ on how they sell season subscriptions, leading to substantial unobserved effects on ticket sales across orchestras; an added benefit of our random-coefficients approach is that it incorporates a random effect that captures any shift in the dependent variable caused by unobservable factors across all events in each individual market, such as the unobservable effect of season subscriptions on single-ticket sales.

The third methodological challenge is that program features are likely to interact requiring the estimation of a large set of pair-wise interactions. We solve this problem by mapping the interactions on a reduced space, arriving at a more parsimonious model formulation. The fourth methodological challenge relates to implementation of the model results beyond the relatively small sample of markets for which historical data were available. To overcome this limitation, we demonstrate how our model can be applied to markets not included in our sample, first using only managerial insight regarding the similarity between the focal market and the ones in our sample and by updating this subjective prior as ticket sales data become available.

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1. Introduction

In 2007, not-for-profit performing arts organizations in the United States garnered revenues of \$5.6 billion, of which private and government contributions accounted for approximately two-thirds (NFAH, 2011). Highly dependent upon governmental and corporate support, these organizations must balance adherence to their mission statements with a difficult financial reality. Music ensembles, opera companies, independent theater companies, and dance companies must plan their performance offerings to meet organizational goals, such as providing community education, continuing a classical tradition, and promoting a new repertoire. At the same time, their “product”

must satisfy attendees as well as donors. Managers of these organizations must sometimes consider demand when making programming decisions to remain financially viable. Attention to demand is becoming even more important as financial support from government and corporations become scarcer.

Symphony orchestras offer a typical example. As in other non-profit arts organizations, ticket revenue accounts for approximately one-third of an average symphony orchestra's gross receipts (LOAO, 2001–2006). Solid ticket sales are therefore a crucial component of any orchestra's financial health. However, assembling a concert season only from popular standard repertoire and film score excerpts would be contrary to the mission statements of most orchestras. The use of statistical methods to more accurately determine the financial ramifications of programming decisions would provide a way of making wiser decisions regarding the concert revenue portion of the budget, enabling music directors to make more adventurous programming choices. Several studies have confirmed the effectiveness of quantitative techniques when applied

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to forecasting for performing arts events (e.g., Bennett, 2002; Weinberg and Schachmut, 1978). Yet, American orchestras are far behind the for-profit sector in adopting analytical techniques for optimizing ticket sales. One reason for this fact may be that orchestra marketing directors are sometimes lacking in the sufficient musical background to plan a financially viable season, while music directors' programming decisions are often informed more by a sense of musical heritage than by any true awareness of the subtleties of audience preferences. Orchestra boards are understandably reluctant to treat "high art" like a consumer product because a slavish attention to market factors would be antithetical to the mission statements of these organizations. However, when implemented in full consideration of these artistic aims, certain methods of econometric modeling could enhance the financial stability of these organizations. As Stephen Belth, Executive Director of the Long Island Philharmonic explains, "If one accepts that concerts should be well attended, and in light of current audience patterns, orchestra organizations must explore every means available to improve long-term attendance of the concert-going public" (Belth, 1999).

Although the task of an orchestra's marketing director is to promote concert events as they are planned by the music director, in actuality, programming decisions are not made by the music director alone. If forecasting tools were made available to a marketing director, perhaps incorporating the orchestra's own historical data, he or she could play a more relevant role in the programming process, helping to fine-tune the season's offerings to achieve a balance between the orchestra's educational and cultural obligations to its community and its own financial wellbeing. One of the roles of the executive director should be to facilitate the inclusion of the marketing director's contributions in the music director's final programming decisions to enhance program appeal to the target audience.

The performing arts have received limited attention in the forecasting literature. Past efforts to build ticket sales forecasting models for performing arts organizations are relatively few in number, and no sources were found in the literature that attempted to model ticket sales for orchestras in particular. Weinberg and Schachmut (1978) developed a practical planning model called "Arts Plan" that used dummy variable regression analysis to predict attendance at a series of music and dance events by taking into account seasonality, product variation, day of performance, number of performances, popularity of offering, and year. In a subsequent article, Weinberg (1986) published a revision of the Arts Plan model which addressed how accuracy might be improved by a manager's knowledge-based modification to a forecast that was based solely on historical data.

In their study modeling Broadway show attendance, Reddy, Swaminathan, and Motley (1998) reported the significance of newspaper reviews, previews, advertising, high-profile actors, and longevity. They found, however, that attendance was relatively price inelastic, echoing similar findings by previous studies of orchestra attendance (e.g., Currim, Weinberg, & Wittink, 1981). Putler and Lele's model (2003) considered market size and venue capacity, and included subjective variables for the number of potential customers and promotional effort, as well as dummy variables for features such as day of week, season, the existence of a competing event, and the "difficulty" of the presentation. In the application of their model to a series of theater productions, the marketing director used model estimates to adjust her own expectations of how each production should be marketed and scheduled. Their model takes into account the number of events in a "run" as well as the possibility of a "sellout," a factor that has not been taken into account in other forecasting models for the performing arts.

Our study differs from previous forecasting models for the performing arts in two important ways. First, our main focus is on understanding how audiences from different orchestral performances respond to the many features of the program for each of multiple concerts, along with their interactions. While previous forecasting models for the performing arts have considered some

program features such as show type, lead performer (Reddy et al., 1998), play type, plot complexity, and social issues addressed (Putler & Lele, 2003), these studies focused on the main effects of a limited number of program features on a single audience or market. In contrast, our proposed model considers potential synergies or negative interactions among the multiple features of a concert program, while also accounting for the fact that different orchestras cater to different audiences, possibly with distinct preferences for certain features of a concert program. Second, because our main interest is in understanding how audiences in different markets react to multiple features of concert programs, we will investigate audience preferences as indicated by single-ticket sales.

Our task presents four major methodological challenges. First, most symphony orchestras in the US have limited historical concert data, which prevents the reliable construction of statistical models at the orchestra level. Therefore, one must leverage available data from all orchestras in the US market. However, the audiences and their preferences for different orchestras are quite distinct, thus requiring a model that accounts for unobserved heterogeneity across audiences using a random-coefficients formulation (Swamy, 1970), a compromise between aggregate and orchestra-level estimation. Unfortunately, preferences for the many (26 in our application) program features are most likely correlated across audiences, requiring the estimation of a large covariance matrix, which would render the typical econometric model over-parameterized for the limited data available. We tackle this particular challenge by capturing the principal components of this large covariance matrix with a formulation that is parsimonious, and therefore more robust for predictions.

Another methodological challenge in uncovering programming preferences for different audiences from ticket sales is the fact that concerts are attended by both single ticket buyers and season subscribers. Furthermore, for many of the orchestras, subscriptions are either unavailable or cannot be directly apportioned to individual events because season subscribers either buy pre-set bundles of events or are granted access to a certain number of events. The fact that orchestras differ in their reliance on subscription sales is likely to create a bias in the estimates of audience preference for program features, unless these (and other) systematic differences across orchestras are taken into account. Our model framework accounts for differences across orchestras through a random-effects formulation which captures the impact of unobservable factors on each market's sales response, such as the fact that these markets differ on the capacity of their venues, and on the percentage of total venue occupancy represented by single-ticket sales.

The third methodological challenge we overcome with our proposed framework is that program features are likely to interact, so that pairing certain program features leads to a concert that is more (or less) attractive than the mere sum of its parts. For example, a guest soloist who is known for his interpretations of the Rachmaninoff Piano Concertos would attract more spectators when performing Rachmaninoff rather than Mozart. The bundling of program features such as soloists, repertoire, and conductor is of great concern to music and marketing directors. Interaction effects have largely been ignored in previous performing arts forecasting models. These feature interactions pose a similar methodological challenge as discussed above because they would require the estimation of a large set ($19 \times 18/2 = 171$ in our application) of pair-wise interactions, rendering the model over-parameterized and thereby less stable for forecasting purposes. We solve this problem by mapping the interactions on a reduced space, arriving at a more parsimonious model formulation.

The final methodological challenge relates to implementation of the model results. Our sample covers only 47 venues for symphony orchestras. While this sample is representative of symphony orchestras in the US, it includes less than 15% of all professional orchestras in the US (LOAO, 2010), which would limit the direct application of our empirical results to only these 47 venues. In other words, while

our results are individually valid for each of the sampled orchestras (as we demonstrate in our predictive validity tests) and generalizable for the population of orchestras, a program director for an orchestra not included in our sample would also want to benefit from insights regarding its own audience, when planning a concert program for his/her orchestra, which requires extrapolation of these results beyond the sample used in our study. To overcome this limitation, we demonstrate how our model can be applied by the music and/or marketing director of an orchestra not included in our sample, first by using managerial insight regarding the similarity between his/her orchestra and the ones included in our sample and later, by updating this subjective estimation as ticket sales data become available for new concerts performed by his/her particular orchestra. With this valuable feature, our model can be used to produce customized insights for other orchestras beyond those we analyzed, as long as its managers can see some similarity between their market and the 47 markets we studied, and/or have sales and program data to tailor our model to their particular market.

In the next section we develop and describe a model that attempts to overcome these four main methodological challenges in uncovering audience preferences revealed through ticket sales. This is followed by a description of our data and the application of the proposed model to them, along with a discussion of the results, predictive validity tests, and implementation issues.

1.1. Modeling audience preferences for program features revealed through concert attendance

Our primary goal in developing our model is to use ticket sales to uncover audience preferences for program features so the results can aid orchestra music and marketing directors in the development of concert programs that achieve their cultural and marketing goals. We fit the model using single-ticket sales as an indicator of audience preferences and test its predictive validity on hold-out forecasts.

As discussed previously, the sample data available from many of the orchestras are limited relative to the large number of features (and pairwise interactions) that must be considered to render the model managerially useful. Moreover, there are several unobservable factors that are idiosyncratic to specific audiences/markets (e.g., season subscriptions, venue size and market potential) that shift observed sales and could lead to biased aggregate estimates of audience response to concert features. For this reason, we must reach a compromise between aggregate (across orchestras) and individual (specific to each orchestra) estimation, which can be achieved with a random-coefficients regression (Swamy, 1970). However, a simple random-coefficients regression that assumes independent random coefficients would be overly simplistic because preferences for program features are most likely correlated across markets, and might also be correlated with unobservable orchestra effects that are captured by the random intercept. On the other hand, as discussed earlier, a full-covariance model would require the estimation of a large number of parameters for the covariance of the random coefficients alone. Again, we reach another compromise between model flexibility and parsimony by capturing the first P principal components (determined empirically) of the covariance among the random regression coefficients (including the intercept), which leads to a more parsimonious and robust formulation. With these considerations in mind, we define our regression model as

$$y_{it} = \sum_k (\beta_k + \sum_{p=1}^P \lambda_{kp} z_{ip}) x_{itk} + \sum_k \sum_{k' > k} \theta_{kk'} x_{itk} x_{itk'} + e_{it} \quad (1)$$

where:

- y_{it} = ticket sales on concert t by orchestra i
- x_{itk} = feature k of the program for concert t by orchestra i , which

- includes a unit vector to capture orchestra-level intercepts
- β_k = average (across orchestras) regression coefficient for feature k (including intercepts, estimated as random effects)
- z_{ip} = score of orchestra i on latent dimension p , independent distributed $N(0,1)$
- λ_{kp} = weight for feature k on latent dimension p
- $\theta_{kk'}$ = coefficient capturing the pair-wise interaction between program features k and k'
- e_{it} = independent distributed random error $\sim N(0,\sigma)$.

The first term in the right-hand side of Eq. (1) shows the principal-components decomposition of the random regression coefficients, where the response coefficient for a specific orchestra i on a program feature k is given by $\beta_k + \sum_{p=1}^P \lambda_{kp} z_{ip}$. Note that these random coefficients include a random-effect (intercept) coefficient for each orchestra, which captures the effects of all factors affecting sales that are unobservable at the audience/market level, such as the fact that orchestras differ on the (unobservable) way they handle season subscriptions. Because the latent scores z are independent standardized normals, the covariance between the random regression coefficients for features k and k' is given by $\sum_{p=1}^P \lambda_{kp} \lambda_{k'p}$, thereby allowing us to capture $K(K+1)/2$ covariance terms with a set of K^*P parameters and maintain parsimony as long as the number of dimensions P is smaller than the number of program features K . Notice that we allow for heterogeneity across markets not only in their response to the program features but also in the intercept. This heterogeneity in the intercept is critical to capturing any unobserved effects that are unique to each market. For example, each of the symphony orchestras in our sample (with the exception of four) performs in a single venue, and these venues vary in size across markets. This variation in venue size, along with differences in potential audiences across markets can inject bias into the estimated response coefficients unless they are taken into account, which is accomplished in our model via the random effects.

The second term in the right-hand side of Eq. (1) captures the possible pair-wise interactions among the K program features. As they are specified in Eq. (1) these interactions require the estimation of $K(K-1)/2$ pair-wise $\theta_{kk'}$, rendering the model over-parameterized for the available data. For this reason, we use another form of space reduction to map the $K(K-1)/2$ interactions into a smaller set of K^*Q coordinates on Q dimensions (to be determined empirically),

$$\theta_{kk'} = \sum_{q=1}^Q \delta_{kq} \delta_{k'q}, \forall k' \neq k, \quad (2)$$

with $\lambda_{kq} = 0$ if $q > k$ for identification purposes.

Notice that we assume homogeneity in the effect of feature interactions on preferences. We do this for two main reasons. First, we assume that interactions are intrinsic to the concert features, and therefore less affected by preferences. In other words, featuring Yo-Yo Ma playing the piano, rather than the cello would seem odd, regardless of the patrons' preferences for the performer or the instruments. Second, we assume homogeneity for practical reasons because estimating random coefficients for all pairwise interactions would lead to a much larger and less feasible set of parameters to be estimated, thereby rendering the model over-parameterized and unstable for forecasting purposes.

1.1.1. Model estimation

Estimation of the proposed model is relatively simple, requiring the combination of the well-known Expectation-Maximization (EM) algorithm (Dempster, Laird, & Rubin, 1977) with non-linear least squares estimation in the M-step (for the estimation of the interactions in Eq. (2)). In the M-step, the latent scores z_i are treated as missing values and replaced by their expectation from the E-step (described next). The model parameters $\Theta = \{\beta, \lambda, \delta, \sigma^2\}$ can then

be estimated by minimizing the sum of squared errors across orchestras and concerts

$$SSE = \sum_i \sum_t \left[y_{it} - \sum_k \left(\beta_k + \sum_{p=1}^P \lambda_{kp} z_{ip} \right) x_{itk} - \sum_k \sum_{k' > k} \left(\sum_{q=1}^Q \delta_{kq} \delta_{k'q} \right) x_{itk} x_{itk'} \right]^2, \quad (3)$$

which we obtain via standard non-linear least-squares estimation.

In the E-step, the p -dimensional latent scores z_i for each orchestra i are obtained, based on the current parameter estimates from the M-step (Eq. (3)), from the following steps:

1. Obtain M draws of p -dimensional vectors of independent and identically distributed (I.I.D.) standard normals v_m , $m = 1, 2, \dots, M$ using a randomized Halton sequence to best approximate the I.I.D. normal distribution (Train, 2003).
2. For each of the M draws v_m , compute the conditional likelihood for each observation $\{y_{it}, X_{it}\}$, given the current parameter estimates $\Theta = \{\beta, \lambda, \delta, \sigma^2\}$, by replacing the unknown heterogeneity factor scores z_i by v_m .

$$L\{Y_{it}, X_{it} | \Theta, v_m\} = \prod_i \prod_t \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2\sigma^2} \epsilon_{imt}^2\right\}$$

where

$$\epsilon_{imt} = y_{it} - \sum_k \left(\beta_k + \sum_{p=1}^P \lambda_{kp} v_{mp} \right) x_{itk} - \sum_k \sum_{k' > k} \left(\sum_{q=1}^Q \delta_{kq} \delta_{k'q} \right) x_{itk} x_{itk'}$$

and v_{mp} is one of the I.I.D. random draws on dimension p .

3. Compute new estimates of the heterogeneity and non-stationary factor scores

$$z_i = \frac{\sum_{m=1}^M [v_m \cdot L(Y_{it}, X_{it} | \Theta, v_m)]}{\sum_{m=1}^M L(Y_{it}, X_{it} | \Theta, v_m)}$$

The expectation and maximization steps described above are repeated until the parameters from the current M-step are reasonably close to those obtained in the previous cycle. The dimensions of the two reduced spaces (P , capturing heterogeneity in preferences across markets, and Q , capturing interactions among program features) are determined empirically by fitting the model with different values of P and Q and comparing their expected predictive fits via the Bayesian Information Criterion (Schwarz, 1978) or Akaike's Consistent Information Criterion.

1.1.2. Implications of the model

Because the proposed model is basically a random-coefficients regression, albeit with a factor structure in the random coefficients and with homogeneous feature interactions, it produces estimates both at the aggregate and orchestra levels. The parameter β_k captures the average preference across all markets for the program feature k , including the intercept. However, the model also accounts for the fact that program features interact, allowing for the possibility that combinations of program features might produce results that are greater than the individual contributions of each feature, while other combinations might interact negatively. The total aggregate appeal of a particular

program, as defined by the features $X = \{x_k, k = 1, 2, \dots, K\}$, can be computed from the model estimates (B, Δ) as

$$V\{X, B, \Delta\} = \sum_k \beta_k x_k + \sum_k \sum_{k' > k} \left(\sum_{q=1}^Q \delta_{kq} \delta_{k'q} \right) x_k x_{k'}. \quad (4)$$

Most importantly, if the music or marketing director for a specific orchestra i with factor scores Z_i (obtained during model calibration) wants to know how the same program might work on her particular audience, the audience-specific appeal can be computed as

$$V\{X, B, \Delta, \Lambda, Z_i\} = V\{X, B, \Delta\} + \sum_k \left(\sum_{p=1}^P \lambda_{kp} z_{ip} \right) x_{itk}. \quad (5)$$

Note that the results in Eq. (5) pertain to a specific audience for orchestra i , thereby taking into account the idiosyncratic preferences in that particular market, but utilizing information from all other markets through shrinkage estimation, which is accomplished by the factor loadings (λ) estimated across all markets and the factor scores z_i obtained for that specific market.

It is important to note that the audience/market-level effects in Eq. (5) include an intercept for market/audience i , which captures the effects of unobservable factors affecting ticket sales for all events for market/audience i . Such factors include the fact that orchestras differ in the (unobservable) way they handle season subscriptions, their venue capacity, and market potential. These unobservable differences across orchestras are captured in the random intercept which, in our factor-analytic formulation, is also allowed to correlate with the observable features of the concerts offered by the orchestra. We believe this feature of the proposed model is particularly important because problems in measuring the dependent variable lead to unobservable differences across orchestras, which must be taken into account for a better assessment of audience preferences. Our proposed formulation accounts for these unobservable differences through random intercepts across orchestras.

2. Empirical illustration

We illustrate and test our proposed model using data collected for 47 symphony orchestra markets in the United States. These orchestras were among those included in the American Symphony Orchestra League's 2004–2005 Orchestral Repertoire Report. At that time, only orchestras with budgets of more than \$1.7 million were included. No subscription ticket data were included, nor were data for complimentary or reduced-rate tickets. Because subscription ticket sales are handled in different ways by each orchestra and are not easily assigned to individual events (and consequently to programming decisions for each concert), our model focuses solely on single-ticket sales. Although orchestras are traditionally dependent on a strong subscriber base, this focus on single-ticket sales also responds to a recent trend toward higher single-ticket sales and lower season subscriptions. Furthermore, some scholars (Johnson & Garbarino, 2001) have argued that the costs of retaining and renewing subscriptions outpace the transaction costs of single tickets, and that successful organizations with too many subscribers may lose revenue on seats that are not sold for the optimal price.

Obviously, because orchestras differ on their reliance on season subscriptions, omission of this and other orchestra-level factors will shift the dependent variable for each orchestra in an unobservable fashion, leading to biased estimates of the sales-response model unless these potential biases are accounted for. However, as described previously (Eq. (5)), our modeling framework directly incorporates adjustments for unobservable differences across orchestras via a random-effects formulation, thereby accounting for these unobserved disparities between orchestras. One especially important and useful feature, given the substantial unobserved differences across markets, is the fact that the proposed modeling framework accounts for these unobservable

effects through random-effects across orchestras (which are also allowed to correlate with responses to observable concert features).

Participating orchestras provided all available single-ticket data, usually several seasons' worth, ranging from 23 concerts on one extreme to 619 concerts at the other, with an average of 110 concerts per orchestra. We also collected data from one orchestra that provided only 15 events, which we used for an out-of-sample illustration, further demonstrating how the results from our sample of 47 orchestras can be generalized for an orchestra not included in our sample.

The dependent variable for our model is log(occupancy), where occupancy is defined as the number of single tickets divided by the venue capacity.² We chose to define our dependent variable as venue-occupancy attributed to single tickets for two main reasons. First, almost all the orchestras in our sample performed in a single venue. Four exceptions performed in two venues, in which case the secondary venue was reserved for distinct programs, such as chamber music. Each of these four exceptions was split into two unique markets. Because each orchestra is uniquely associated with a single venue, differences in venue size are directly captured by the random effect, as discussed earlier, thereby justifying the use of occupancy as the dependent variable. Second, we did not have access to the number or proportion of seats taken by season subscribers in each concert, which we believe varies across orchestras. This is also captured by the random intercepts because we define our dependent variable as the proportion of seats occupied by single-ticket buyers. Although it would have been desirable to include the number of season tickets assigned to each concert in our dependent variable, this information, along with complimentary or reduced-price tickets, was not available to us. We see this lack not only as a data limitation but also as a practical problem that makes our random-effects formulation particularly valuable in this illustration because it accounts for unobservable differences across orchestras.

We constructed the following dummy variables for use as predictors in the model:

- | | |
|---|---|
| <ul style="list-style-type: none"> • Saturday evening • Sunday afternoon • Sunday–Thursday evening • January, February, March • April, May, June • September, October • Star Soloist • Soloist with 20 or more appearances in sampled concerts • Guest conductor? • Includes at least one “most popular” piece • Includes at least one “other popular” piece | <ul style="list-style-type: none"> • Choral (other than popular) • Mozart (other than popular) • Beethoven (other than popular) • Tchaikovsky (other than popular) • Brahms (other than popular) • R. Strauss • Prokofiev • Other Works (pre-Romantic) • Other Works (Romantic) • Other Works (late Romantic) • Other Works (turn of the century) • Other Works (twentieth century) • Contemporary (alive) |
|---|---|

Based on discussions with orchestra marketing directors, we ascertained that seasonality was a relevant factor in ticket sales. To preserve parsimony, we categorized each concert into one of four “seasons” rather than into months with November–December used as the basis. To compile a list of star soloists, we aggregated the Billboard data from 1977 to 2005 (the top fifteen albums of each month's report). Soloists who appeared more than 6 times in the Top Classical Albums (Billboard, 2005a) charts in the twenty years prior to a given concert were considered “star soloists,” as were soloists who appeared more than 6 times in the Top Crossover Classical Albums (Billboard, 2005b) charts in the ten years prior to a given concert. Because some famous soloists focus more on live performances than on recording, we also included a variable to indicate if a concert program featured a soloist who made 20 or more “appearances” in the sampled data. Some conductors are especially popular and draw large audiences when they

are in town (e.g., Michael Tilson Thomas in San Francisco); conversely, some conductors are famous enough that they will attract a large audience when they guest conduct (e.g., most recently Gustavo Dudamel). We attempted to capture this effect with a “guest conductor” variable.

The works being performed play a large role in most concertgoers' decisions to attend (John & Knight Foundation, 2003, Table B1). Because only 6% of potential classical consumers consider themselves “very knowledgeable” about classical music (50% call themselves “not very knowledgeable”), and because concerts with familiar music are preferred, it follows that there is a small collection of musical works that would significantly increase revenue and attendance if included on a program. We expected that concert programs which include better-known works would have a strong tendency to attract bigger audiences and that the “popularity” of a given work would have a stronger positive correlation with ticket revenue than any other repertoire-related criterion (e.g., nationality, composer, genre). For this reason we established the categories “most popular” and “other popular.” Because six in ten orchestra ticket buyers listen to classical music on the radio daily or several times a week, we used a review of eight “classical countdown” lists assembled by various radio stations nationwide (as of April 2006) to determine the prevailing tastes of concertgoers. Works that were in the top 28 of all eight lists were deemed “most popular,” and works which were in the top 28 of at least two (but no more than seven) of the lists were assigned to the “other popular” category. Choral works are known to draw bigger audiences, even if they were not “popular,” so we established a separate choral work category. Works in this category did not include “popular” works, which were already slotted to the “most popular” and “other popular” categories.

In a recent study, of the 43% of potential classical consumers who indicated that they had a favorite composer, 23% named Beethoven, 23% Mozart, 15% Bach, 9% Tchaikovsky, and 4% Chopin (John & Knight Foundation, 2003). While the tastes of actual concertgoers are almost certainly more diverse, these data suggest that the inclusion of music by certain renowned composers on a particular program would increase both attendance and revenue. This is also substantiated by the fact that people tend to prefer music with which they are somewhat familiar (John & Knight Foundation, 2003). Therefore, because the works of Mozart, Beethoven, Tchaikovsky, Brahms, Richard Strauss, and Prokofiev were sufficiently represented in the data, their works were assigned their own dummy variables. Popular works, however, were excluded from these categories (for example, for the purposes of our study, Beethoven's Ninth Symphony was considered a “Most Popular” work rather than a work by Beethoven). Works by other composers were categorized by era. Pre-Romantic composers include those born before 1800, Romantic composers include those born between 1800 and 1849, Late Romantic composers include those born between 1850 and 1873, Turn-of-the-Century composers include those born between 1873 and 1899, Twentieth Century composers include composers who were born in the Twentieth Century who are no longer alive, and Contemporary (“Alive”) composers include all living composers. Several less commonly found subcategories (music of Igor Stravinsky and Maurice Ravel, and music of Twentieth Century American composers) are omitted from the categorization and comprise the default value. Table 1 presents summary statistics for all the variables used in our study.

We constructed a “Quality-Adjusted Price” measure from the inflation-adjusted average ticket price for each concert. This calculation was performed to account for the fact that prices might be endogenously determined based on program costs. For example, a program featuring a *star soloist* such as Yo-Yo Ma or a commissioned piece by a contemporary composer will have higher costs, and might also be more appealing to the audience. Therefore, the marketing directors may wish to recover the higher costs with higher than average ticket prices for the event.

To the best of our knowledge, this potential endogeneity has been ignored in the past marketing literature in the performing arts, which may explain the low price elasticities reported in those studies. To

² We compared our dependent variable (log-occupancy) with a logistic transformation, and found virtually perfect correlation (0.996) between the two across a rather large sample of events, which strongly suggests the two DV's will produce very similar predictions and insights.

Table 1
Data summary.

Variable	Minimum	Maximum	Mean	Variance
Log(ticket sales/capacity)	−5.403	−0.234	−1.890	0.377
Saturday evening			0.340	0.224
Sunday afternoon			0.128	0.112
Sunday–Thursday evening			0.250	0.187
January, February, March			0.363	0.231
April, May, June			0.247	0.186
September, October			0.208	0.165
Quality adjusted price	−28.530	58.625	0.066	140.077
Star soloist			0.080	0.073
Soloist with 20 or more appearances			0.257	0.191
Guest conductor			0.337	0.223
Includes at least one “most popular” piece			0.073	0.068
Includes at least one “other popular” piece			0.300	0.210
Choral (other than popular)			0.100	0.090
Mozart (other than popular)			0.143	0.123
Beethoven (other than popular)			0.096	0.087
Tchaikovsky (other than popular)			0.083	0.076
Brahms (other than popular)			0.093	0.085
Strauss			0.075	0.070
Prokofiev			0.066	0.062
Other works (pre-Romantic)			0.150	0.128
Other works (Romantic)			0.347	0.227
Other works (late Romantic)			0.318	0.217
Other works (turn of the century)			0.324	0.219
Other works (twentieth century)			0.240	0.183
Contemporary (alive)			0.274	0.199

adjust prices for programming quality, we ran the regression reported in Table 2 across all events and for all orchestras, and utilized the regression residuals as the quality-adjusted prices, assuming that the cost structure for these program features is common across orchestras. Notice that we only adjusted prices for features that are directly related to program content, while the sales model (to be discussed later) was estimated on all event features, which ensures the identification of both models. As one would expect, *star soloist* and *soloist with more than 20 concerts* show a positive and statistically significant coefficient indicating that these features tend to be offered at higher prices across orchestras. Featuring a *guest conductor* was not found to positively affect ticket prices, while the inclusion of popular pieces tended to have a negative effect on ticket prices, most likely because these pieces are associated with regular events sold at normal (lower) average ticket prices. Most importantly, the residuals from this hedonic price regression provide a more valid measure of ticket prices that is adjusted for the extra cost associated with special features.

Another potential form of endogeneity or source of supply-side effects is the possibility that program/music directors might build and schedule programs to best match the expected audiences to optimally occupy their venue. Although this calculation is possible in theory, requiring a simultaneous supply/demand model across all orchestras and concerts with the proper instruments to disentangle the two sides, in practice, program/music directors operate under numerous unobservable (to the researcher) constraints. For example, some orchestras do not own their venues, in which case venue availability is of primary importance. The availability of an orchestra's music director is also paramount. In some cases, orchestras need to alternate their regular season Classical subscription series with a Pops series, which means that the availability of the Pops conductor and the Pops performers must be taken into account as well. Smaller and mid-size orchestras are often required to contract their performers on a weekly or per-service basis, further limiting scheduling flexibility. In many smaller and mid-size markets, orchestral players perform and rehearse with other orchestras in neighboring cities (e.g., Portland, ME

Table 2
Hedonic price regression on program content.

Program content	Unstandardized coefficients	Beta	Statistical significance
(Constant)	36.66	0.00	
Star soloist	6.49	0.14	0.00
Soloist with 20 or more appearances	1.12	0.04	0.00
Guest conductor	0.60	0.02	0.09
Includes at least one “most popular” piece	−0.70	−0.02	0.30
Includes at least one “other popular” piece	−2.43	−0.09	0.00
Choral (other than popular)	−0.27	−0.01	0.63
Mozart (other than popular)	−0.79	−0.02	0.11
Beethoven (other than popular)	−1.90	−0.05	0.00
Tchaikovsky (other than popular)	−1.78	−0.04	0.00
Brahms (other than popular)	−2.32	−0.06	0.00
Other works (pre-Romantic)	−2.26	−0.07	0.00
Other works (Romantic)	−3.02	−0.12	0.00
Other works (late Romantic)	−2.17	−0.08	0.00
Other works (turn of the century)	−1.47	−0.06	0.00
Other works (twentieth century)	−1.63	−0.06	0.00
Contemporary (alive)	−2.56	−0.09	0.00
Strauss	0.73	0.02	0.29
Prokofiev	−0.21	0.00	0.77

Note: Values in **bold** are statistically significant at the 0.05 level.

and Providence, RI). Particular star soloists who are in demand may or may not be available on a particular date, and their inclusion in the calendar will also affect the other components of the program. Given these and other complexities, as well as the practical difficulties in finding proper and relevant instruments, we leave a more comprehensive demand/supply equilibrium model for future research. It should be noted, however, that we estimate individual orchestra effects via the random-coefficients formulation, which captures any endogeneity bias at the orchestra level.³

2.0.1. Empirical results

To test for the predictive validity of our proposed model, we held out data from the last five concerts from each of the 47 orchestras in our calibration sample so we could compare the forecasts with actual ticket sales for these five future concerts in true step-ahead forecasts. To demonstrate how the results can be generalized beyond our sample of orchestras, we held out the data from one of the smaller orchestras, so that the results obtained from the other 47 orchestras could be used to produce out-of-sample predictions for the orchestra that was excluded from the calibration sample.

As we discussed previously, the proposed model allows for a flexible accounting of heterogeneity in preferences for program features across audiences, and also of the interactions among program features, while maintaining parsimony by capturing the principal components of the covariance matrix of random preferences and by space reduction of the pair-wise interactions. However, this parsimony is determined by the choice of dimensions to account for heterogeneity and interactions. To guide our decisions for each dimension, we fit the proposed model under various configurations. Several fit statistics are reported in Table 3.

Table 3 compares the fit for the aggregate model ignoring differences across audiences without interactions (first column) and with interactions mapped onto one or two dimensions (next two columns). As one would expect, there is a small improvement in fit by

³ We also ran a fixed-effects model as an alternative approach to handle endogeneity. We obtained fixed effects that are statistically indistinguishable from the individual-level estimates obtained with our formulation, which supports our model specification.

allowing for feature interactions, indicated by a shift in R^2 from 21.2% to 23.9% and 24.6%. This is confirmed by Akaike's Information Criterion (AIC), but not by the Consistent AIC (CAIC), which imposes a more stringent penalty for extra parameters, proportional to the logarithm of the sample size. The improvement in expected predictive performance by allowing for heterogeneity in preferences across audiences is quite clear, particularly for the models with one and two latent factors, in which all indicators favor the more complex formulation. However, the CAIC suggests that the 3-factor solution is over-parameterized, relative to the two-dimensional factor model. In other words, the CAIC criterion indicates that adding more parameters to capture unobserved heterogeneity (i.e., adding more complexity to the covariance matrix of the random coefficients) would cost more, in terms of complexity, than it is worth in terms of predictive fit. In the extreme case, a full specification of the covariance matrix would lead to an even more over-parameterized model (our attempts to estimate this fully saturated model failed to converge).

The AIC criterion suggests that the modeling of feature interactions would improve predictive performance, while the more stringent CAIC does not support the inclusion of these interactions. Because the interactions between program features provide potentially valuable insights to music and marketing directors, we will report the detailed results for the complete model with two latent-factors for audience heterogeneity and two dimensions capturing feature interactions. Later, we will compare the forecasting performance for all versions of the model listed in Table 3.

One potentially important consideration ignored by our model and implemented here, is that if the same type of concert is played over and over again at the same venue, there is likely to be a saturation effect in the sense that attendance will go down. To test the demand-side implication of this phenomenon, we expanded the model with variables representing saturation effects (Van Oest, van Heerde, & Dekimpe, 2010). We did this by adding a lagged binary variable indicating whether the same feature was offered in either of the past two previous concerts in the same venue, creating 18 additional binary predictors for each feature of the concert program (other than scheduling and price). However, this inclusion represented a heavy burden in terms of additional parameters to be estimated (54 additional parameters for a 2-dimensional heterogeneity map, assuming that the saturation effects do not interact). A model comparison in terms of expected predictive validity (measured by CAIC) clearly showed that incorporating the 18 additional saturation predictors led to an over-parameterized model. In other words, the additional predictors did improve fit (as one should expect), but the loss of degrees of freedom was not justified by the improvement in fit,

therefore jeopardizing the (predictive) validity of the extended model.⁴ We took these results as indication that preference saturation is not a major limitation to attendance in our particular application.

Table 4 shows the parameter estimates for the complete model with two latent factors accounting for heterogeneity in preferences across audiences and two dimensions capturing the interactions among program features, where parameters that are statistically significant at $p < 0.01$ are marked in bold characters. For a more intuitive interpretation of the typical effect of program features on single-ticket occupancy, we report in Table 5 the expected average percentage changes in single-ticket occupancy in response to a 10% reduction in quality-adjusted price and with the inclusion of each of the other program features across all orchestras and concerts.

As one would expect, on average (shown in the first column) all markets respond positively to *Star Soloist*, *Most Pop Piece*, *Other Pop Piece*, and *Choral*, while responding negatively to prices (after correcting for program content). On average, audiences also respond positively to *Mozart* pieces that are not among the most popular, but respond negatively to less popular works – and works by lesser-known or less popular composers – from the *Pre-Romantic*, *Romantic*, *Late Romantic* and *20th Century* periods. For example, the inclusion of masterworks such as Dvořák's Eighth Symphony and Sibelius' Violin Concerto, both of which were among the most frequently performed works in the 2005–2006 season, on a program apparently does not translate into ticket sales, perhaps because they are not as well known to the general public.

However, the model suggests that audiences do not respond more negatively to the inclusion of a *Contemporary* work on the program, than to the inclusion of less-known pieces from the *Pre-Romantic* or *Romantic* periods. The reason for this is unclear, but there are several possibilities. Contemporary works can allow for additional marketing possibilities (e.g., the composer is local, the work is about a popular contemporary figure, etc.). Furthermore, recent trends in orchestral composition favor the incorporation of elements from rock and popular music, making contemporary music increasingly accessible to a broader public. And occasionally, film composers – the most famous of contemporary composers – are included on concert programs. Howard Shore's *Lord of the Rings Symphony*, for example, was a great financial success in Des Moines and Spokane. On average, audiences also seem to prefer the resident conductor over a *Guest Conductor*. The number of “star” conductors seems to be low when compared to the number of star soloists; guest conductors are perhaps less well-known to audiences and may be harder to market than the local personality.

The latent factor coefficients (second and third columns of estimates in Table 4), when combined with the factor scores for each orchestra, determine how the preferences of the audience of each orchestra depart from the averages discussed above, while also accounting for systematic differences in single-ticket occupancy across audiences/markets via the intercept. Even though the factor coefficient and respective factor scores were obtained in a very different way (they were obtained as the principal components of the unobservable multivariate distribution of regression coefficients) than standard factor analysis (which is performed directly on observed variables), these estimates are interpreted in a similar way. These factor coefficients are more meaningful when displayed in a vector map, which is shown in Fig. 1, and can be interpreted as factor loadings for the respective regression coefficients. In this map, program features are represented as vector *termini* pointing to the orchestra positions (defined by the orchestra factor scores) that respond better than average to these program features. For example, the darker vector pointing to *intercept* indicates that orchestras positioned

Table 3
Fit statistics for different formulations of the model.

Model	Fit Statistics				
	Log-likelihood	Parameters	AIC	CAIC	R2
Aggregate	–3990.0	26	8032	8201	21.2%
Aggregate + 1D Interactions	–3958.0	45	8006	8299	23.9%
Aggregate + 2D Interactions	–3933.9	63	7994	8404	24.6%
1D Random Coefficients	–2811.0	51	5724	6056	52.0%
2D Random Coefficients	–2639.5	77	5433	5935	55.2%
3D Random Coefficients	–2546.1	102	5296	5961	56.8%
2D Random Coeff + 1D Interactions	–2607.0	96	5406	6031	55.6%
2D Random Coeff + 2D Interactions	–2573.1	114	5374	6117	56.4%

Note: AIC = Akaike Information Criterion.
CAIC = Akaike Consistent Information Criterion.
R2 = % of variance explained in the dependent variable by the model.

⁴ We also tested a simpler and more parsimonious model with a common saturation effect for all program features, but obtained a saturation effect that was not statistically significant at the 0.05 level.

Table 4
Parameter estimates for the complete model with two latent factors for audience heterogeneity and two dimensions capturing feature interactions.

Feature	Average	Audience Heterogeneity		Feature Interactions	
		Factor 1	Factor 2	Interact1	Interact2
Intercept	-2.125	0.950	-0.084	-	-
Sat eve	0.182	-0.169	0.002	-	-
Sun after	0.003	-0.116	-0.097	-	-
Sun–Thu eve	-0.141	-0.343	-0.191	-	-
Jan, Feb, Mar	0.018	0.090	0.027	-	-
Apr, May, Jun	-0.053	0.164	0.040	-	-
Sept, Oct	-0.146	0.060	-0.011	-	-
Price	-0.014	0.040	0.020	-	0.005
Star soloist	0.142	0.120	0.108	-0.206	-0.298
Freq soloist	-0.081	-0.029	-0.038	-0.715	-0.150
Guest conductor	-0.122	0.021	0.029	-0.039	0.128
Most pop piece	0.424	-0.071	0.123	0.057	0.341
Other pop piece	0.242	-0.067	0.084	0.046	-0.081
Choral	0.226	-0.203	-0.057	0.313	0.359
Mozart	0.083	-0.190	-0.116	-0.055	-0.092
Beethoven	-0.042	0.174	0.099	-0.114	0.037
Tchaikovsky	0.057	-0.113	-0.011	-0.091	-0.035
Brahms	-0.023	-0.301	-0.211	-0.151	0.179
Strauss	-0.190	0.058	-0.041	-0.099	0.154
Prokofiev	-0.088	0.178	0.195	-0.075	-0.077
Pre-Romantic	-0.119	-0.227	-0.208	-0.258	0.048
Romantic	-0.130	-0.025	-0.064	-0.118	0.149
Late Romantic	-0.169	-0.087	-0.058	-0.222	0.368
Turn of century	-0.032	-0.192	-0.147	-0.059	0.017
20th century	-0.095	-0.034	-0.085	0.000	0.000
Contemporary	0.008	-0.163	-0.010	0.024	-0.102

Values in bold are statistically significant at the p<0.01 level.

further in that direction have larger than average intercepts, and therefore tend to occupy a higher percentage of their venues with single-ticket buyers, all else being the same. These differences in intercepts across orchestras also account for unobservable factors such as season subscriptions that systematically cause single-ticket occupancy differences across markets. Orchestras located in the opposite direction have lower than average single-ticket sales as a proportion of seating capacity. This finding could be due to such reasons as a venue that is too large for the market, a large base of season-ticket holders, or strong

Table 5
Average single-ticket occupancy elasticities for program features.

Program feature	% change in occupancy rate
Saturday evening	13.3
Sunday afternoon	-2.8
Sunday or Thursday evening	-21.1
Jan, Feb, Mar	5.0
Apr, May Jun	0.4
Sept, Oct	-11.6
10% cut in quality-adjusted price	0.4
Star soloist	19.3
Popular soloist	-6.8
Guest conductor	-11.0
Most popular piece	45.7
Other popular piece	22.6
Choral	11.0
Mozart	3.8
Beethoven	3.5
Tchaikovsky	3.7
Brahms	-7.0
Pre-Romantic	-11.0
Romantic	-9.3
Late Romantic	-9.2
Turn of century	-5.8
20th century	-9.4
Contemporary	-7.2
Strauss	-9.5
Prokofiev	-2.2

competition from other forms of entertainment in the same market. We see this ability to incorporate unobservable differences across orchestras/markets as a valuable feature of our proposed model for capturing audience response for each orchestra. Most of the program features point in opposite directions along the same line, as indicated by the light-color vector. The program features that seem to distinguish the audiences the most are *Prokofiev, Most Pop Piece, Star Soloist, Beethoven* and *Other Pop Piece* in one direction and *Brahms, Sun-Thu Eve, Turn of Century* and *Mozart* in the other direction.

Considering that the preference heterogeneity map in Fig. 1 shows the locations where audiences are more/less responsive than average to program features, it would be useful to know where each of the 47 audiences is located in this same map. The locations of the 47 audiences in the same preference heterogeneity map are shown in Fig. 2. Combining Figs. 1 and 2 we can now see that orchestras positioned in the far right tend to occupy their venues with single-ticket sales at a higher rate than other orchestras, while those on the far left have lower than average occupancy with single-ticket sales, again due to many unobservable factors (the remaining seats may be either empty or occupied by season subscribers; we do not have data to identify them). Nearly all of the orchestras from large metropolitan areas are located in the right half of Fig. 2; it is reasonable to assume that orchestras in large markets might be better able to fill their venues with single tickets as opposed to season tickets or unsold seats. Furthermore, several of the orchestras shown in the left part of the graph are based in smaller cities but have very large venues and are not likely to fill closer to capacity with single-ticket sales only.

Figs. 1 and 2 suggest that audiences in the top right quadrant of Fig. 2 respond more favorably to *Prokofiev, Star Soloist, Most Pop Piece, Other Pop Piece, Beethoven (other than popular)*, and also buy more single tickets relative to the total capacity of the venue because they are located farther away from the origin along the direction indicated by the intercept vector in Fig. 1. With the possible exception of *Prokofiev*, these variables represent program features that are more marketable (e.g., “non-popular” Beethoven, which actually has a negative intercept). Orchestras in the top right quadrant may have a more effective non-subscriber marketing strategy or an advantage due to market size. However, several larger orchestras with high single-ticket ratios apparently respond comparatively less favorably to popular program features. These orchestras seen in the lower right quadrant of Fig. 2. One potential explanation for this might be that star soloists and popular classical music offerings are less unique for large markets. The true reason for these correlations with the random intercepts is unknown because these intercepts capture all unobservable factors affecting the general level of single-ticket sales (across events) for each orchestra.

The audience factor scores shown in Fig. 2, combined with the coefficients shown in Fig. 1, provide some insights into the program features that are more likely to produce higher single-ticket sales for each particular audience. For example, CHT and NYP have the largest scores on Factor 1 and therefore are further along in the intercept direction (see Fig. 1), which means that these markets tend to occupy their venues with a higher than average proportion of single tickets. These two orchestras also have some of the lowest scores in Factor 2, suggesting that their audiences respond better than average to *Brahms, Turn of Century*, and *Pre-Romantic*, but worse than average to star soloists and popular works. This could mean that the single-ticket audiences for these orchestras are more devoted or knowledgeable about classical music, or that the marketing for “popular” concerts is less aggressive than it would be in other cities. In contrast, DET and PIT also occupy more of their venues (relative to the average orchestra) with single-ticket buyers (positive scores on Factor 1), but they are located further in the vector directions for *Prokofiev, Beethoven* and *Star Soloist* (see Fig. 1), suggesting that their audiences respond better than average to these program features.

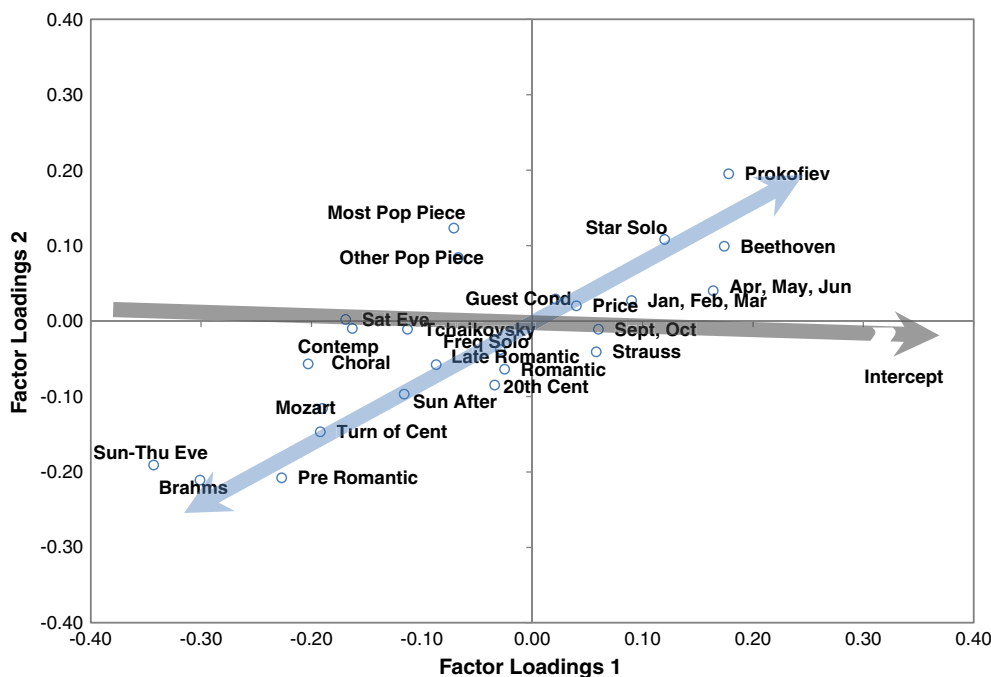


Fig. 1. Map of preference heterogeneity. Note: Points represent vector termini indicating the direction where an orchestra would have a larger than average response coefficient for the respective feature. For example, the direction denoted by the truncated horizontal vector indicates where orchestras would have a larger than average intercept.

As discussed previously, one must also consider that program features may interact with each other, so that the total impact of a program can be higher or lower than the sum of the individual effects, depending on whether the interactions are positive or negative. The 171 (= 19 × 18/2) pairwise interactions among the 19 program features are parsimoniously captured by the 37 coordinates in a 2-dimensional space, shown in the last two columns of Table 4. As with the factor coefficients, these coordinates are more meaningful when displayed in a map in which each program feature is represented by a vector. This interaction map is displayed in Fig. 3.

The interaction map in Fig. 3 summarizes the pattern of interactions among the program features, so that features pointing in the same direction interact positively, while those pointing in opposite directions interact negatively. For example, features such as *Turn of Century* and *Price* have short vectors in Fig. 3, and therefore do not interact with other features. On the other hand, the interaction map shows *Star Soloist* pointing in the opposite direction to *Most Pop Piece*, indicating a negative interaction, while *Freq Soloist* interacts positively with *Star Soloist* because their vectors point in the same direction.

The interactions implied by the interaction map in Fig. 3 are reported in Table 6, where one can see a few statistically significant interactions. For example, Table 6 confirms the negative interaction between *Star Soloist* and *Most Popular piece*, suggesting that people who are attracted to a concert by a star soloist may tend to be the same who would be attracted by the most popular pieces. On average both features produce more single-ticket sales, but the inclusion of both features on the same program does not result in an additive effect. The same is true of the features *Choral* and *Star Soloist*. It is possible that choristers' friends and family tend to purchase single tickets, but it may be true that they are also attracted to the concert hall when a well-known star soloist is also on the program. Combining these two program features produces an overall effect that is less than the sum of the two individual effects because the audience attracted to the concert hall by *Choral* overlaps with the audience attracted by *Star Soloist*.

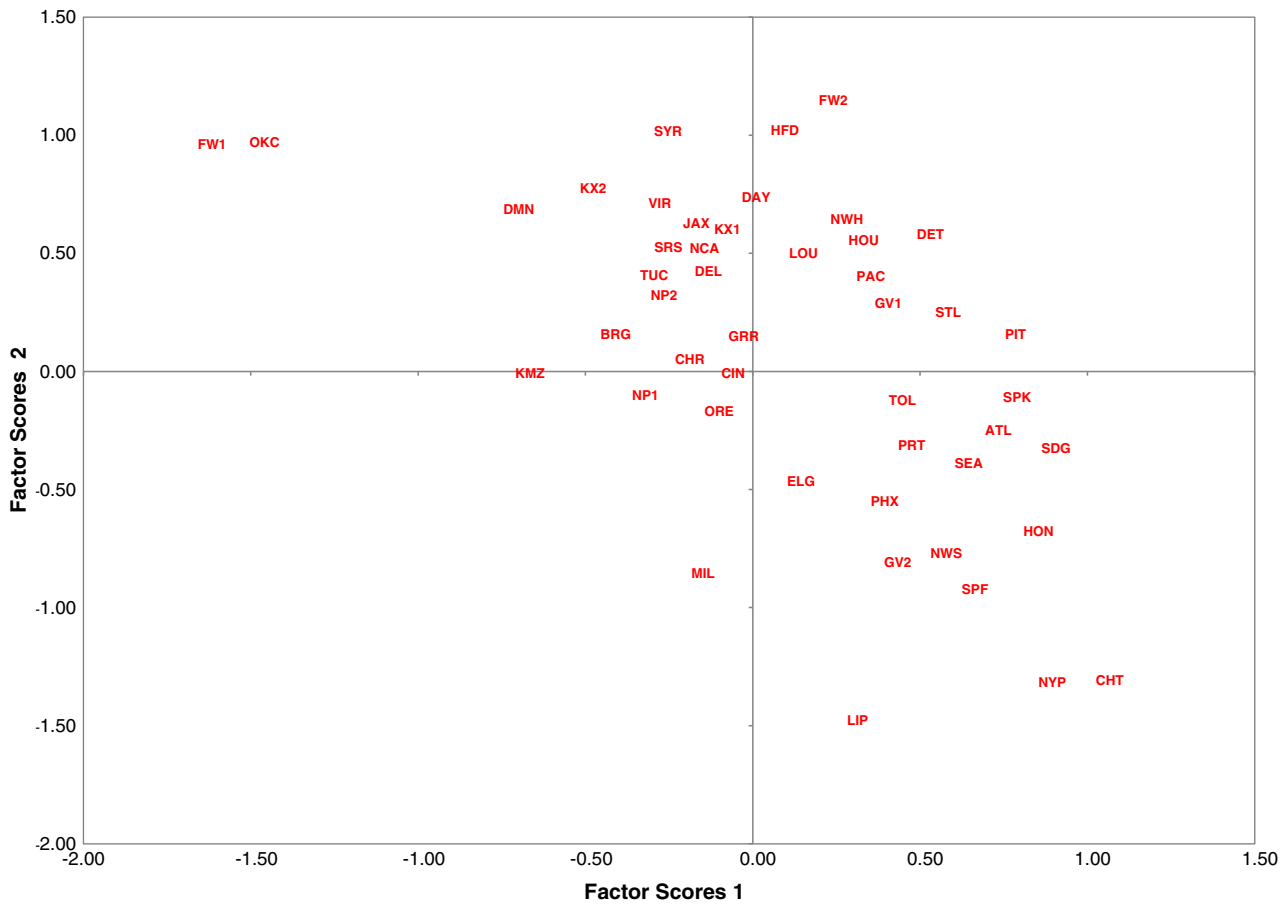
On the other hand, the positive interaction between *Star Soloist* and *Freq Soloist* suggests that featuring a star soloist who has also been frequently featured in past concerts produces a better result.

In other words, this positive interaction suggests that for soloists, familiarity leads to audience loyalty. It is also interesting to note that the slightly negative average effects of *Romantic* music, *Late Romantic* music, and the neutral average effect of the music of *Brahms* on a program (see Table 3) show a positive interaction when these repertoires are combined (Fig. 3 and Table 6), suggesting that while each of these items have a neutral or negative average effect, combining them tends to improve the overall attractiveness of the program.

2.0.2. Predictive validity tests

Even though the main goal in developing the proposed model was to obtain a better understanding of audience preferences for concert features, we test for the predictive validity of the proposed model and verify the consequences of over-specification. We apply the parameter estimates obtained from the calibration sample to predict log-occupancy for the last five concerts for each of the 47 orchestras that were withheld from the dataset. Obviously, there are many factors affecting ticket sales for a concert (beyond the program features and scheduling information included in our model), such as competing entertainment options at the same time in the same market, weather conditions at the time of the event, and marketing efforts to promote the season series. Therefore, the predictive performance of this model should be viewed with its main purpose of uncovering preferences for concert features in mind rather than for producing accurate ticket sales forecasts. Despite these caveats, the predictive fit results we discuss next are quite reasonable and strongly support the predictive validity of our proposed framework. Moreover, they show considerable improvement with our proposed framework relative to simpler models that ignore the various aspects captured by our model.

The fit statistics shown in Table 7 confirm that accounting for heterogeneity across audiences has a considerable effect on model performance. Overall, the best predictive performance is obtained with two factors accounting for preference heterogeneity across audiences and one dimension uncovering the interactions among program features. As one would expect, there is some degradation in fit between model calibration and holdout forecasts. However, the



Orchestra Key for Preference Heterogeneity Map (for figure 2)

ATL Atlanta	HON Honolulu	ORE Oregon
BRG Baton Rouge	HOU Houston	PAC Pacific (Orange Cty., CA)
CHR Charlotte	JAX Jacksonville	PHX Phoenix
CHT Chattanooga	KMZ Kalamazoo	PIT Pittsburgh
CIN Cincinnati	KX1 Knoxville (Venue 1)	PRT Portland (ME)
DAY Dayton	KX2 Knoxville (Venue 2)	SDG San Diego
DEL Delaware	LIP Long Island	SEA Seattle
DET Detroit	LOU Louisville	SPF Springfield (MA)
DMN Des Moines	MIL Milwaukee	SPK Spokane
ELG Elgin	NCA North Carolina	SRS Santa Rosa
FW1 Fort Wayne (Venue 1)	NP1 Northeast Penn. (Venue 1)	STL Saint Louis
FW2 Fort Wayne (Venue 2)	NP2 Northeast Penn. (Venue 2)	SYR Syracuse
GRR Grand Rapids	NWH New Haven	TOL Toledo
GV1 Greenville (Venue 1)	NWS NewWorld (Miami)	TUC Tucson
GV2 Greenville (Venue 2)	NYP New York	VIR Virginia
HFD Hartford	OKC Oklahoma City	

Fig. 2. Location of each market/venue in the preference heterogeneity map.

performance in both samples is reasonably close, suggesting stability in the model specification.

2.0.3. Implementation for a non-sampled orchestra

The discussion above focused on the insights the model provides regarding the average appeal of a particular concert program, and on deviations in preferences around this average for the 47 orchestras in our sample. However, the results from our proposed model can also be utilized for an orchestra not included in that sample. Without

any prior information regarding how her audience might depart from the average, the marketing director for this orchestra can use the average estimates, reported under “intercept” and “feature interactions” in Table 4, which provide insights into how a “typical” or average audience would respond to the program being considered, as shown in Eq. (4).

On the other hand, if the marketing director believes there are some similarities between her audience and some of the 47 orchestras portrayed in Fig. 3, she could use this information to guess the

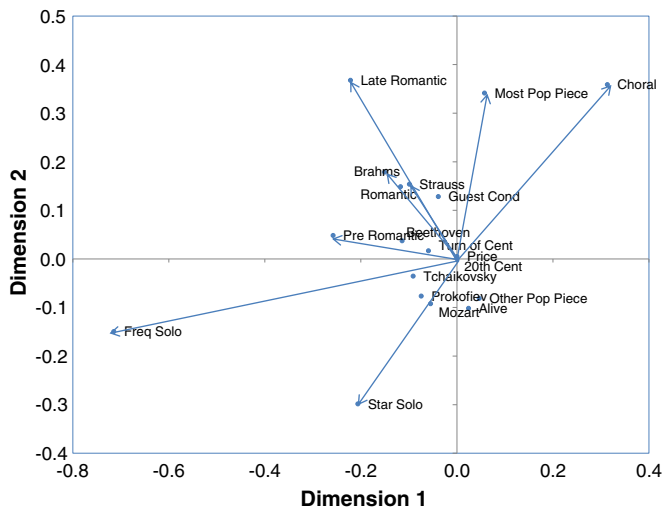


Fig. 3. Interaction map. Note: points represent vector termini indicating the strength (length) and direction of each feature in the interaction map. Features pointing in the same (opposite) direction have a positive (negative) interaction with magnitude proportional to the length of their vectors.

likely factor scores for her audience using the scores (map positions) for the audiences in Fig. 3 she believes are similar to her own. This “guesstimate” of the scores for her audience would produce estimates of a program’s appeal that are more customized to her particular audience (utilizing Eq. (5)) than the average appeal from Eq. (4). As single-ticket sales data become available for her orchestra, the marketing director can obtain data-driven estimates of the factor scores for her audience following the calculations we describe for the E-step right after Eq. (3).

As an illustration, we consider an anonymous orchestra for which we have data on 15 concerts, not included in our calibration sample. First, we produce forecasts based on the preferences of the typical audience. Then, we attempt to customize the forecasts based on our own subjective (admittedly naive) assessment of the similarities between this orchestra and the ones in our calibration sample. Finally, we use ticket sales data from the first 3, 6 and 9 concerts to obtain factor scores for this new orchestra, which we then use to produce forecasts for the remaining periods. Table 8 and Fig. 4 compare the forecasting performance obtained with the proposed model under these different conditions. From this figure and table one can see that the forecasting performance is substantially improved as sales data become available to compute the audience’s latent scores, which show how this audience departs from the average across all orchestras in the calibration sample. Once the scores for the new orchestra are better estimated, the forecasting performance of the model stabilizes at a much lower prediction error than the *a priori* forecasts.

3. Conclusions and final discussion

The forecasting model developed and tested in this study illustrates how an organization that produces events by combining interacting features can learn about its audience’s preferences and, in this process, predict ticket sales with greater accuracy. Non-profit performing arts organizations in particular can benefit from our approach by minimizing risk associated with scheduling performers, determining the content and context of these performances, commissioning contemporary pieces, and recruiting guest artists while meeting organizational goals. Following our proposed framework, a similar model could be developed for independent theater companies, dance companies, or opera companies, that would produce insights into their audience’s tastes, while

Table 6
Pairwise program-feature interactions implied by the proposed model.

	Price	Star solo	Pop solo	Guest cond	Most pop piece	Other pop piece	Choral	Mozart	Beethoven	Tchaikovsky	Brahms	Pre Romantic	Romantic	Late Romantic	Turn of cent	20th cent	Alive	Strauss	Prokofiev
Price	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Star solo	0.00	0.17	0.19	-0.03	-0.12	0.01	-0.17	0.04	0.01	0.03	-0.02	0.04	-0.02	-0.06	0.01	0.00	0.03	0.00	0.00
Pop solo	0.00	0.19	0.68	0.01	-0.09	-0.02	-0.28	0.05	0.08	0.07	0.08	0.18	0.06	0.11	0.04	0.00	0.00	-0.02	0.04
Guest cond	0.00	-0.03	0.01	0.03	0.04	-0.01	0.04	-0.01	0.01	0.00	0.03	0.02	0.02	0.06	0.00	0.00	0.00	0.05	0.07
most pop piece	0.00	-0.12	-0.09	0.04	0.16	-0.02	0.14	-0.03	0.01	-0.02	0.05	0.00	0.04	0.11	0.00	0.00	-0.01	0.02	-0.01
Other pop piece	0.00	0.01	-0.02	-0.01	-0.02	0.01	-0.01	0.00	-0.01	0.00	-0.02	-0.02	-0.02	-0.04	0.00	0.00	0.01	-0.02	0.00
Choral	0.00	-0.17	-0.28	0.04	0.14	-0.01	0.29	-0.05	-0.02	-0.04	0.02	-0.06	0.02	0.06	-0.01	0.00	-0.03	0.02	-0.05
Mozart	0.00	0.04	0.05	-0.01	-0.03	0.00	-0.05	0.02	0.00	0.01	-0.01	0.01	-0.01	-0.02	0.00	0.00	0.01	-0.01	0.01
Beethoven	0.00	0.01	0.08	0.01	0.01	-0.01	-0.02	0.00	0.03	0.01	0.03	0.03	0.02	0.04	0.01	0.00	-0.01	0.02	0.01
Tchaikovsky	0.00	0.03	0.07	0.00	-0.02	0.00	-0.04	0.01	0.01	0.02	0.01	0.02	0.02	0.08	0.01	0.00	0.00	0.00	0.01
Brahms	0.00	-0.02	0.08	0.03	0.05	-0.02	0.02	0.03	0.03	0.01	0.08	0.05	0.04	0.10	0.01	0.00	-0.02	0.04	0.00
Pre Romantic	0.00	0.04	0.18	0.02	0.00	-0.02	-0.06	0.01	0.03	0.02	0.05	0.09	0.05	0.08	0.02	0.00	0.01	0.03	0.02
Romantic	0.00	-0.02	0.06	0.02	0.04	-0.02	0.02	-0.01	0.02	0.01	0.04	0.04	0.05	0.08	0.01	0.00	-0.02	0.03	0.00
Late Romantic	0.00	-0.06	0.11	0.06	0.11	-0.04	0.06	-0.02	0.04	0.01	0.10	0.08	0.08	0.24	0.02	0.00	0.04	0.08	-0.01
Turn of cent	0.00	0.01	0.04	0.00	0.00	0.00	-0.01	0.00	0.00	0.00	0.01	0.02	0.01	0.02	0.01	0.00	0.00	0.01	0.00
20th cent	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
Alive	0.00	0.03	0.00	-0.01	-0.03	0.01	-0.03	0.01	-0.01	0.00	-0.02	-0.01	-0.02	-0.04	0.00	0.00	0.02	-0.02	0.01
Strauss	0.00	-0.02	0.05	0.02	0.05	-0.02	0.02	-0.01	0.02	0.00	0.04	0.03	0.03	0.08	0.01	0.00	-0.02	0.07	0.00
Prokofiev	0.00	0.04	0.07	-0.01	-0.03	0.00	-0.05	0.01	0.01	0.01	0.00	0.02	0.00	-0.01	0.00	0.01	0.00	0.00	0.03

Statistically significant at the 0.05 level.

Table 7
Predictive validity tests.

Model	Predictive fit		
	MSE	Variance	R-square (%)
Aggregate	0.44	0.51	14.5
Aggregate + 1D interactions	0.45	0.51	12.6
Aggregate + 2D interactions	0.45	0.51	14.1
1D Random coefficients	0.29	0.51	43.3
2D Random coefficients	0.29	0.51	44.2
3D Random coefficients	0.28	0.51	45.3
2D Random coeff + 1D interactions	0.27	0.51	46.7
2D Random coeff + 2D interactions	0.29	0.51	44.4

Note: MSE = mean squared error.

accounting for interactions among program features and for preference heterogeneity across markets. Using a non-repertoire-specific set of variables, the model could be useful for venues that mix genres and performance types, such as civic centers or community performance spaces. And, with a modified dependent variable, exhibition spaces and museums could employ a version of our model as well. Because it accounts for interactions among program features, our proposed framework allows the organization to anticipate the potential synergies produced by combining certain features or the potential cannibalization of audience resulting from adding a certain feature to a program. For example, a contemporary art museum might want to know the ideal balance of artists to display. Should the museum purchase a second Damien Hirst or add the first Ai Wei Wei to its collection? Understanding these interactions across other contemporary museums would help in this type of decision. Because it allows for heterogeneity in preferences across markets via a factor-regression random-coefficients formulation, our framework allows each organization to “borrow” information from similar organizations, leading to more robust insights about the organization’s own audience and a more reliable assessment of the interactions among program features. A successful performance season at a performing arts center in Dallas might differ dramatically from a successful mix of offerings at a performing arts center in Boston.

Specifically, this study provides insights that could be useful to an orchestra’s music and marketing directors about general audience preferences for concert features. Although some of the program features in the study were, as expected, positively or negatively correlated with single-ticket occupancy (see Table 4), a few surprising general conclusions about orchestra single-ticket sales may be drawn from the data:

- the inclusion of a soloist with frequent engagements does not typically result in higher occupancy,
- the inclusion of a guest conductor on the program has a significant negative impact on occupancy,
- less popular works by famous composers such as Beethoven, Brahms, and Tchaikovsky do not necessarily translate into higher attendance,
- less popular works from before the turn-of-the-century have a stronger negative effect on occupancy than less popular works from after the turn-of-the-century, and perhaps most surprisingly,
- contemporary music is the only category of less popular works that does not have a significant negative effect on single-ticket occupancy.

The more predictable results of the model include the positive effects of choral music and popular and famous works and soloists on occupancy, the low occupancy rate that typically occurs at the beginning of a season (September–October), the popularity of Saturday evening, and the lower occupancy that occurs on weeknights.

Table 8
Predicted single-ticket sales (as a % of seating capacity) for a new orchestra.

Concert	Actual	Forecasts				
		No prior	Subjective prior	After 3 concerts	After 6 concerts	After 9 concerts
1	33%	13%	12%			
2	18%	21%	20%			
3	19%	14%	11%			
4	23%	14%	12%	22%		
5	26%	16%	14%	23%		
6	14%	13%	10%	16%		
7	18%	19%	18%	29%	30%	
8	18%	15%	12%	18%	18%	
9	20%	13%	10%	19%	19%	
10	24%	15%	12%	18%	18%	18%
11	18%	12%	11%	18%	18%	17%
12	19%	17%	15%	22%	22%	21%
13	20%	13%	10%	16%	16%	16%
14	18%	14%	12%	20%	20%	19%
15	32%	17%	15%	24%	25%	23%
	MAPE	30%	41%	15%	16%	16%

Note: MAPE is the Mean Absolute Percentage Error computed for the last 6 concerts. ‘No Prior’ represents predictions based on the average orchestra. ‘Subjective prior’ represents predictions using a subjective prior based on perceived similarity. ‘After x concerts’ represents predictions after updating the model parameters using available data.

The interaction effects shown in Fig. 3 are also informative for marketing and music directors. The interaction effects of chief interest are:

- the positive interaction between choral works and the most popular pieces, implying that the joint effect of including a choral work and a “most popular” piece on the same program is greater than the sum of their individual effects;
- the negative interaction between works featuring a star/frequent soloist and popular or choral works, implying that the joint effect of including a star/frequent soloist on the same program as a popular or choral work is less than the sum of their effects; and
- the positive interactions between many of the characteristics of the orchestra’s less popular core repertoire (Pre-Romantic, Romantic, Late Romantic, Brahms, Strauss, Beethoven), suggesting that the combination of these types of works is more effective than the inclusion of a single work of this type on a program.

Our empirical results suggest several programming strategies for music directors who are concerned with maintaining steady single-ticket sales. The interaction effects observed in our model imply that occasional concertgoers are more likely to attend a program that has both a choral work and a very popular piece than a program that has only one or the other. The pairing of works in these categories may result in an overall increase in single-ticket sales. Furthermore, concertgoers who are inclined to attend a program featuring a star (or frequent) soloist are often the same concertgoers who are inclined to attend a program featuring a choral or popular work. Including a star soloist on a program that already includes a choral work or a popular work will not significantly increase single-ticket sales. Finally, combining less popular works (from 1750 to 1950) together on the same concert may (counter-intuitively) be the best way to maximize single-ticket sales for programs of this type. Such a program would, however, tend to sell fewer tickets than a program with a popular work included.

The average effects have additional implications for the business-minded marketing director. For example, single-ticket buyers seem to not be interested in attending performances that feature a guest conductor. It would be wiser, perhaps, to emphasize the resident conductor’s role as a figure in the community and increase sales by marketing him or her as a personality. This approach has achieved success in some markets (Ankeny and Neeme, 2005). Additionally, the inclusion of a featured soloist on a program does not necessarily translate into higher single-

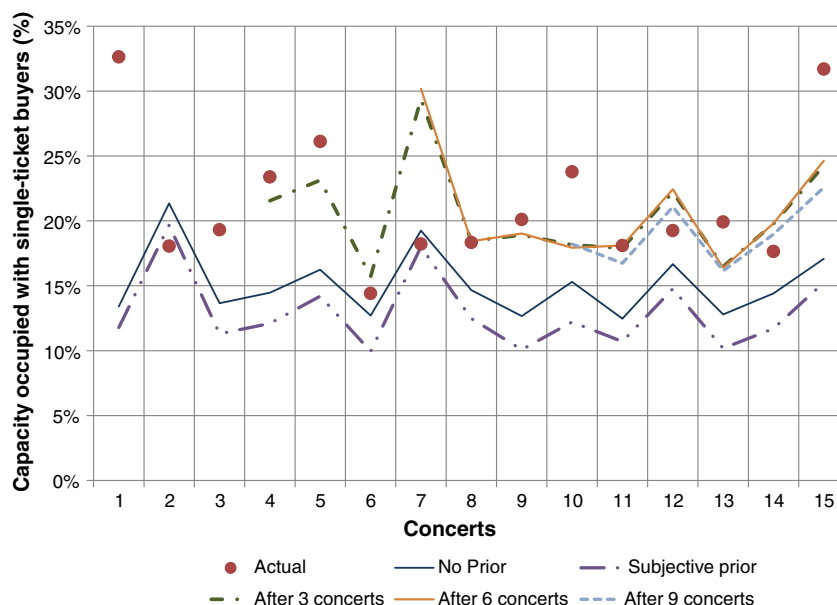


Fig. 4. Predicted single-ticket sales (as a % of capacity) for a new orchestra based on prior and updated estimates.

ticket sales, even if that soloist has numerous engagements with other orchestras. Only star soloists (i.e., chart-topping recording artists) are likely to draw additional single-ticket buyers. Finally, single-ticket buyers seem to be less interested in works from before 1900 – even those considered to be standard repertoire – unless the work can be considered a “popular” piece. Related to this phenomenon is the non-negative effect of contemporary compositions on single-ticket sales. If marketed in the right way, contemporary music has the potential to draw larger audiences of single-ticket buyers than Romantic Era music. Just as single-ticket buyers seem to prefer and identify with their local conductor, they may more easily identify with composers of today (especially local composers) than with composers of two hundred years ago.

It is clear from Fig. 2, however, that significant variation in preference exists across markets. This figure, combined with Fig. 1 provides direct insights into how each audience differs from the sample average, helping the music director to tailor a concert program to the specific tastes of her audience. Most importantly, as we demonstrated in our empirical illustration, managers from any orchestra included in our sample can obtain parameter estimates that reflect the tastes of their specific audience. Additionally, as demonstrated in the empirical illustration, managers from orchestras not included in our sample may utilize the model by locating their audience in Fig. 2 based on their similarity to other audiences or by gathering historical data on concert features and ticket sales for their own orchestra.

In this study, we focused on single-ticket sales because they are more likely to depend on the features of each concert, as evidenced by the fact that most participating orchestras were unable to provide subscription data at the concert level. It should be noted that this is a limitation of the data available to us rather than of the proposed model. In fact, our random-coefficients formulation attempts to minimize the biasing effect of these unobservable season tickets by incorporating a random component capturing unobservable orchestra-level effects. Future applications could extend our proposed forecasting framework to include season ticket sales, which would have to be broken down to the event level so they can be directly related to program features.

Because we did not have data on subscription sales for each event, we also ignored the possibility that some events might have been sold-out of single tickets, something that has usually been overlooked in the performing-arts literature, with the notable exception of Putler and Lele (2003). Our sources in the industry indicate that sold-out regular-season concerts are uncommon. However, as data on

all ticket sales become available for each event, a truncated extension of our proposed framework will be necessary to account for this truncation in ticket sales due to venue capacity constraints.

As noted previously, our effort focused on how audiences respond to program features, assuming these features as exogenous variables and ignoring the possibility that music/program directors have already developed an intuition about their audiences' preferences and behaviors, using this intuition to design and schedule programs to maximize utilization of their venues. Therefore, program features could be endogenous to a supply–demand system, rather than exogenous as we assumed in our demand model. The real problem is much more complex than what one would expect in theory because music/program directors must operate under many external constraints. Of course, to engage a highly coveted soloist or guest conductor, scheduling adjustments may be necessary. More importantly, most orchestras do not own their own venues, and they must first construct the season's schedule subject to the availability of the hall. Additionally, the music director must be available for most or all of the concerts. If the orchestra has a Pops series, the regular season Classical subscription series must be scheduled so as not to conflict with the Pops series. The availability of the Pops conductor and performers would then need to be taken into account as well.

The availability of the orchestra's players is also crucial, and this varies from orchestra to orchestra because larger orchestras have salaried players who are expected to attend all rehearsals and performances, whereas smaller and mid-size orchestras have players under contract for a certain number of weeks or services. This means that for most orchestras, the addition of an extra service in a given week can be prohibitively expensive, or requires the elimination of a service elsewhere in the season's schedule. Furthermore, in many smaller and mid-size markets orchestral players perform and rehearse with other orchestras in nearby cities (e.g., Portland, ME and Providence, RI), so schedules are sometimes constructed to allow the musicians additional performance opportunities. In Portland, for example, there is a long tradition of Tuesday night concerts because that is when players happen to be available.

Unfortunately, we could not find any historical register of the context in which programming and scheduling decisions were made by the 47 orchestra directors (with as many as 619 concerts each). A retrospective re-construction of this record would be impractical and fraught with errors and inconsistencies. Therefore, modeling programming/scheduling decisions and audience response simultaneously was not feasible with the available data. Nevertheless, a more comprehensive

equilibrium model that would simultaneously account for the demand and supply decisions made by all audiences and all planners would be a logical next step, as more comprehensive data are gathered from the supply side. We leave this as an opportunity for future research.

Finally, although our focus in this study was on the performing arts and symphony orchestras in particular, the main features of our proposed framework can be potentially useful in many other marketing contexts in which managers must assemble multiple, potentially interacting attributes to attract customers who may respond differently to these attributes in different markets. Our framework allows the manager to uncover the diversity in “tastes” for product/service features across markets while accounting for potential synergy and cannibalization among these features. We hope the framework will also prove useful and insightful for marketers offering different bundles of features in multiple markets (e.g., fast-food chains, grocery retailers and hotel chains).

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