British Journal of Mathematical and Statistical Psychology (2003), 56, 215–229

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The structure of self-reported emotional experiences: A mixed-effects Poisson factor model

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Multivariate count data are commonly analysed by using Poisson distributions with varying intensity parameters, resulting in a random-effects model. In the analysis of a data set on the frequency of different emotion experiences we find that a Poisson model with a single random effect does not yield an adequate fit. An alternative model that requires as many random effects as emotion categories requires high-dimensional integration and the estimation of a large number of parameters. As a solution to these computational problems, we propose a factor-analytic Poisson model and show that a two-dimensional factor model fits the reported data very well. Moreover, it yields a substantively satisfactory solution: one factor describing the degree of pleasantness and unpleasantness of emotions and the other factor describing the activation levels of the emotions. We discuss the incorporation of covariates to facilitate rigorous tests of the random-effects structure. Marginal maximum likelihood methods lead to straightforward estimation of the model, for which goodness-of-fit tests are also presented.

1. Introduction

The problem of analysing multivariate count data arises frequently in applied research. One such example, which motivated the research in this paper, is the study of the relationship among frequencies of different emotion experiences. As suggested by common observation, people differ in their susceptibility to pleasant and unpleasant emotions. In fact, the investigation of the determinants of such individual differences is a focus of much recent research in personality and clinical psychology (Diener, 1999; Russell & Barrett, 1999). In this paper, we analyse data from a diary study of 128 respondents who were asked to report how often they experienced different emotions (Schimmack, 1996). In addition, the participants in this study answered a battery of questionnaires that included the NEO Personality Inventory (Costa & McCrae, 1992).

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The data can be summarized in the form of a matrix of independent variables (personality measures) and a frequency table with rows representing the respondents and columns the various emotion categories. The focus of our analysis is to determine both the dependency structure of the columns and relationships between this dependency structure and the independent variables.

Whereas multivariate methods for the analysis of associations in rating data are readily available (Kryzanowski, 1990), comparable methods for multivariate count data are less well developed. Perhaps the most popular model for analysing multivariate count data is the negative multinomial distribution (Guo, 1996; Land, McCall, & Nagin, 1996; Thall, 1988; van de Ven & Weber, 1995; van Duijn & Böckenholt, 1995). This model allows for a common distribution of the means of the Poisson variables and thus assumes a positive equicorrelation structure among the counts. Aitchison and Ho (1989) present a more general model by postulating a multivariate lognormal distribution to describe the joint variation in the category-specific rate parameters. However, despite its generality, this approach has received limited attention in the literature because high-dimensional numerical integration is required for the estimation of the model parameters, limiting severely the number of event categories that can be considered.

Recently, Bartholomew and Knott (1999) and Moustaki (1999) proposed latent-trait models for mixed-outcome variables in the exponential family. This approach appears to be well suited for studying dependencies among multivariate count data because it provides a graphical representation of the data which can facilitate greatly the substantive interpretation of the dependence structure. As shown below, the latent-trait approach can be viewed as an extension of the Aitchison and Ho (1989) model. Building on this work, we present a multivariate, two-level Poisson factor model with item-specific random effects. The first level is defined by a latent-trait model which relates the incidences of the various emotion categories to a set of common and unique latent factors via a log-link function. On the second level, the random factor scores are regressed on the personality measures. As a result, we obtain a parsimonious representation of how the different emotion categories relate to each other, and we can study the extent to which this structure can be explained by a set of covariates. Marginal maximum likelihood methods are used for estimation which simplifies inference and goodness-of-fit issues.

2. A mixed-effects Poisson factor model

Consider an $N \times J$ matrix of counts cross-classified by respondents i and items j. The Poisson probability function for count y_{ij} is

$$\mathrm{P}(y_{ij} \mid \theta_{ij}) = \frac{\exp(-\theta_{ij})\theta_{ij}^{y_{ij}}}{y_{ij}!}. \tag{1}$$

When modelling the dependency structure of the counts y_{ij}, it is useful to take into account the effects of two sets of explanatory variables which are specific either to respondents and items, $\mathbf{x}_{ij}$, or to respondents alone, $\mathbf{z}_i$. Using the log-link function, we relate $\mathbf{x}_{ij}$ to the Poisson mean for respondent i and item j by

$$\ln(\theta_{ij}) = \mathbf{x}_{ij}'\alpha_j + \gamma_j'\omega_i, \tag{2}$$

where the $L \times 1$ vector $\mathbf{x}_{ij}$ includes the intercept component, and γ_j contains the unknown factor 'loadings' of item j. The fixed regression effects of item j are denoted by

α_j, and the random factor effects by ω_i. We relate the factor coefficients to a $K \times 1$ vector of person-specific covariates, $\mathbf{z}_i$, by

$$\omega_i = \mathbf{B}\mathbf{z}_i + \epsilon_i, \tag{3}$$

where $\mathbf{B}$ is a $P \times K$ matrix of regression coefficients and ϵ_i follows a P-variate normal distribution. Combining these two levels of the model and substituting $\gamma_j^* = \gamma_j'\mathbf{B}$, we obtain

$$\theta_{ij} = \exp(\mathbf{x}_{ij}'\alpha_j + \mathbf{z}_i'\gamma_j^* + \gamma_j'\epsilon_i). \tag{4}$$

We refer to (4) as the mixed-effects Poisson factor model. Although the assumption of a standard multivariate normal distribution for ϵ could be relaxed to allow for correlated random effects, it is not possible to identify this correlation structure. For any non-singular transformation matrix $\mathbf{T}$, $\gamma_j'\epsilon_i$ can be rewritten as $\gamma_j'\mathbf{T}\mathbf{T}^{-1}\epsilon_i$. We also note that alternative distributional assumptions for the random effects or different link functions are possible. For example, if all factor weights are positive, it may prove useful to apply an identity link with a multivariate gamma distribution for the random effects (McCullagh & Nelder, 1989; Fahrmeir & Tutz, 2001).

Equation (4) includes several special cases. When the factor loadings γ_j are known, (4) simplifies to a two-level Poisson regression model (Diggle, Liang, & Zeger, 1994; Goldstein, 1995; Hedeker, 1998). Setting $\gamma_j = 1$ for all j yields a multivariate version of Hinde's (1982) Poisson-normal regression model. Finally, (4) becomes a regression version of Aitchison and Ho's (1989) lognormal model if the number of factors is equal to the number of items, $P = J$. However, we expect that in many applications the number of factors required to capture dependencies among the observed categories is considerably smaller than the number of event categories, which simplifies both the interpretation and estimation of the random-effects structure. To investigate the effects of factors that are specific to each item, we later present an extension of (4) that allows for 'uniqueness' terms and thus facilitates a direct test of (4).

2.1. Estimation

Marginal maximum likelihood methods are used for the estimation of the mixed-effects Poisson factor model. Under random sampling of N respondents, the log-likelihood function can be written as

$$\ln L = \sum_{i=1}^{N} \ln\left(\int_\epsilon \prod_{j=1}^{J} \Pr(y_{ij} \mid \theta_{ij}) g(\epsilon)\, \mathrm{d}\epsilon\right), \tag{5}$$

where $g(\epsilon)$ is a p-dimensional standard normal density function, and θ_{ij} is given by (4). The log-likelihood function (5) is evaluated by multivariate Gauss–Hermite quadrature. Tables for the weights and nodes of this integration rule can be found in Stroud and Secrest (1966). The numerical approximation of the integrals becomes arbitrarily accurate as the number of nodes for each dimension is increased. However, because the number of nodes increases exponentially with the value of p, numerical integration works efficiently only up to five or six dimensions.

Model parameters are estimated by a quasi-Newton method that approximates the inverse Hessian according to the Broyden–Fletcher–Goldfarb–Shanno update (see Gill, Murray, & Wright, 1981). For computational convenience, no restrictions are imposed on the factor weights in the estimation algorithm. However, when standard errors are desired for the factor weights, we impose the constraint that the elements of the lower triangular factor weight matrix are zero. This approach constrains $P(P-1)/2$ parameters in the loading matrix, which is sufficient to address translation, rotation, and scale indeterminacies (Wedel & Kamakura, 2001). The algorithm utilizes the first-order derivatives of the log-likelihood function similarly to Hedeker and Gibbons (1994) who use Fisher's method of scoring for solving the likelihood equations of their random-effects ordinal regression model. The estimation procedure has been implemented in a computer program written in GAUSS 3.5 (Aptech Systems, 1995).

2.2. Model tests

Because of the maximum likelihood framework, it is straightforward to derive specific tests for nested hypotheses. For instance, the significance of the relationship between person-specific covariates and factor scores can be assessed by a likelihood-ratio (LR) statistic (referred to as G^2) that is asymptotically distributed as a χ^2 statistic under the null hypothesis. However, LR tests cannot be applied for comparing models with P and $P+1$ factors because these models are not nested. Although it is possible to test a $P < J$ factor model against the $P = J$ model, it is well known that this LR test is oversensitive to small departures from the null hypothesis because of the large number of degrees of freedom involved (Akaike, 1987; Burnham and Anderson, 1998, p. 62). Equally important, the $P = J$ Poisson factor model may be difficult to estimate because of the high dimensionality of the integration and the large number of parameters.

We consider three approaches to determining the number of factors to be included in (4). First, we consider a more general formulation of the factor model which includes a specific factor for each item. These 'uniqueness' terms can capture additional sources of variation that are not represented by the common factors. Second, we apply information criteria to select the appropriate factor model. Third, we use diagnostic tests for generalized linear models (McCullagh & Nelder, 1989). These tests are also used to examine the overall fit of the Poisson factor model. These approaches are now discussed in turn.

Item-specific overdispersion

A more general version of (4) is obtained by including item-specific random effects. The extended model can be written as

$$\theta_{ij}^* = \exp(\mathbf{x}'_{ij}\alpha_j + \mathbf{z}'_i\gamma_j^* + \gamma'_j\epsilon_i + \phi_{ij}^*), \tag{6}$$

where the ϕ_{ij}^* are item-specific factor weights that are independently distributed for $j = 1, \ldots, J$. A tractable form of (6) is obtained under the assumption that $\phi_{ij} = \exp(\phi_{ij}^*)$ follows a gamma distribution with expected value equal to 1 and index parameter η_j (Cameron & Trivedi, 1998),

$$f(\phi_{ij}) = \frac{\eta_j^{\eta_j}}{\Gamma(\eta_j)}\phi_{ij}^{\eta_j - 1}\exp(-\eta_j\phi_{ij}).$$

Under this specification, the integral for the random item-specific effects can be written in closed form as a negative binomial distribution. Consequently, the probability function for y_{ij} is

$$\Pr(y_{ij}|\theta_{ij}, \eta_j) = \frac{\Gamma(\eta_j + y_{ij})}{\Gamma(\eta_j)y_{ij}!}\left(\frac{\eta_j}{\eta_j + \theta_{ij}}\right)^{\eta_j}\left(\frac{\theta_{ij}}{\theta_{ij} + \eta_j}\right)^{y_{ij}}, \tag{7}$$

where θ_{ij} is given by (4). By comparing the Poisson factor model with the negative binomial factor model, we can determine whether it is necessary to allow for item-specific overdispersion effects.

Information criteria

Although Akaike (1987) argued for what became known as the Akaike information criterion (AIC) to compare models with a different number of factors, it should be borne in mind that this statistic does not asymptotically indicate the true model among a set of candidate models (Bozdogan, 1987). In response, several authors have proposed dimension-consistent criteria with penalty terms for the number of estimated parameters d, such as the Bayesian information criterion (BIC) given by $\mathrm{BIC} = -2\ln L + d\ln(JN)$ (Schwarz, 1978) and the similar corrected AIC statistic (Bozdogan, 1987). Based on the assumptions that model dimensionality is fixed as $N \rightarrow \infty$, and that the true model is among the set of candidate models, these statistics indicate the true model with probability one, asymptotically. We therefore base our model selection on the BIC statistic, which tends to result in more parsimonious models than the AIC statistic. Support for this choice is presented in the next section, which summarizes the results of a simulation study.

Residual analyses

Diagnostic tests for generalized linear models can be applied to investigate the adequacy of the Poisson factor model (4). For example, local misfit can be assessed on the basis of standardized residuals $\hat{q}_{ij} = (y_{ij} - \hat{y}_{ij})/\hat{s}_{ij}$, where $\hat{y}_{ij} = \mathbf{x}'_{ij}\hat{\alpha}_j + \mathbf{z}'_i\hat{\gamma}^*_j + \frac{1}{2}\Sigma^P_{p=1}\hat{\gamma}^2_{jp}$ and $\hat{s}^2_{ij} = \hat{y}_{ij} + \hat{y}^2_{ij}(\Sigma^P_{p=1}\hat{\gamma}^2_{jp} - 1)$. Similarly, multivariate residuals may be computed by

$$\hat{\mathbf{q}}^*_i = \hat{\mathbf{R}}_i^{-1/2}\hat{\mathbf{q}}_i,$$

where $\hat{\mathbf{R}}_i$ is the estimated correlation matrix with elements

$$\hat{r}_{ijk} = \frac{\hat{y}_{ij}\hat{y}_{ik}(\Sigma^P_{p=1}\hat{\gamma}_{jp}\hat{\gamma}_{kp} - 1)}{\hat{s}_{ij}\hat{s}_{ik}}.$$

When the estimated mean rates are small, we also found it useful to compare the observed and expected frequencies for the one- and two-way margins of the data. This approach is illustrated in the context of the application.

3. Results of a simulation study

A small-scale simulation study was conducted to assess both the performance of the numerical integration approach and the hit rate of the BIC statistic in determining the appropriate number of factors. One hundred replications of ten-dimensional count data for $N = 200$ respondents were generated under a one-, two- and three-dimensional Poisson factor model. For each of these three different factor models, the parameter values of both the intercept terms and the factor loadings were specified to be $\pm.5$. Poisson factor models with from one to four factors were fitted to each of the 300 data sets generated.

The BIC statistic identified correctly the true number of factors for the 300 data sets. Although this perfect hit rate may not hold in general, these results strongly support the choice of the BIC statistic, especially when the loadings among the factors are of similar size. Table 1 displays the biases and standard errors of the estimated parameter values. The numbers reported were obtained by averaging the absolute deviations of the ten mean estimates of the intercepts and the ten mean estimates of the loadings for each of the factors from their true values of $\pm .5$. The corresponding averaged standard deviations of the parameters are given in parentheses. As expected, both the bias and the size of the standard errors increase slightly with the number of estimated model parameters. Overall, however, the simulation results demonstrate that the estimation approach is both feasible and promising: the bias is small for the one-, two-, and three-factor models and the loadings appear to be estimated with satisfactory precision.

Table 1. Bias and standard errors of Poisson factor models

	Estimated factor models		
Parameters	1-Factor	2-Factor	3-Factor
Intercept	.00 (.07)	.00 (.09)	.00 (.09)
Factor I	.01 (.07)	.01 (.11)	.01 (.10)
Factor II		.01 (.11)	.01 (.11)
Factor III			.02 (.12)

Note: The entries of the table are the average absolute bias and the standard errors (in parentheses) of the intercept parameters and the parameters of the first, second and third factor (rows) under a one-, two- and three-factor model (columns), respectively.

4. Application to the Affect Study

As suggested by common observation, individuals tend to experience momentary events consistently in a positive or negative manner (Diener, 1984; DeNeve & Cooper, 1998). Although causes are not well understood, there is agreement that personality traits at least partially determine this behaviour (McCrae & Costa, 1991; Watson & Clark, 1992). In recent research two personality factors, extraversion and neuroticism, have been hypothesized to be directly associated with current pleasant and unpleasant affective states. Neuroticism assesses proneness to experience unpleasant emotions and emotional lability. Extraversion assesses sociability, enthusiasm and pleasure arousal. Neuroticism and extraversion appear to be correlated with intense unpleasant and pleasant emotions, respectively (Bachorowski & Braaten, 1994; Schimmack & Diener, 1997).

The investigation of the relationship between emotional experiences and personality traits has been hampered by a lack of consensus on their underlying structure. Russell and Barrett (1999, p. 805) note in their literature review that ‘some researchers use categories, some dimensions; some use bipolar concepts, some unipolar ones; and some presuppose simple structure, some a circumplex, and some a hierarchy’. Recently, however, Yik, Russell, and Barrett (1999) conjectured that many of these representations can be captured by a dimensional structure defined by two bipolar dimensions that represent the pleasantness and activation level of emotions (see also Larsen & Diener, 1992). ‘Sad-happy’ is an example of the pleasant-unpleasant dimension, while ‘aroused-quiet’ is an example of the high-low activation dimension. ‘Anxious-euphoric’ represents unpleasant-pleasant affects with a high level of activation, while 'drowsy-content' represents unpleasant-pleasant affects with a low level of activation.

Clearly, the notion of a two-dimensional structure of emotions can serve only as a first approximation. Russell and Barrett (1999) proposed an extension of this framework by introducing the distinction between 'core affect' and 'prototypical emotional episodes'. According to this conceptualization, a person is always in some state of 'core affect', defined by feelings of activation and pleasure. In contrast, 'prototypical emotional episodes' may involve discrete emotions that have a different structure. Examples include fighting someone in rage or watching an accident in terror. Emotion words do not follow this conceptual distinction between 'core affect' and 'emotional episodes'. For example, a person may be 'relaxed' because she has just finished a period of meditation (an emotional episode) or for no apparent reason. This ambiguity can introduce instabilities in factor-analytic representations of self-reported affects since the presence of 'emotional episodes' may distort relationships among 'core affects'. As a result, an investigation of the dimensionality of the 'core affect' space requires repeated measures to assess the degree of instability in empirically derived factor-analytic solutions.

In the empirical study reported here, our focus is thus twofold. The first question of interest is whether the underlying structure of 'core affects' is consistent with Yik *et al.*'s (1999) conjecture of a two-dimensional model. Our approach differs in two important aspects from previous studies on this issue. First, we analyse daily self-reports of affect separately over a period of 14 days to determine the variability in factor-analytic solutions which is caused presumably by the presence of 'emotional episodes'. Second, our self-report data are collected in the form of counts. In contrast, previous studies on the dimensionality of the affect space relied almost exclusively on rating scales for self-reports of affect frequencies. By using counts, we can avoid the methodological problems inherent in rating scales that are caused by individuals differing in scale usage and interpretation of the frequently arbitrary number of response categories and labels. The second question of our investigation concerns the relationship between the derived affect structure and the two personality factors, extraversion and neuroticism. To our knowledge, this relationship has never been investigated directly. Clearly, a regression of the factor coefficients on the personality variables provides a testable and rigorous representation of associations between personality and affect.

4.1. Data

Our investigation of the structure of emotions and its relationship to the personality factors neuroticism and extraversion is based on a diary study conducted by Schimmack (1996). The data set consists of 128 respondents who were asked to report daily over a period of 21 days how often they experienced 20 different emotions. Ten of these emotions represented reactions to particular events (e.g. jealously, envy) and were omitted from the analyses. The remaining ten emotions were less episode-driven and classified as pleasant (joy, euphoria, contentment, relief and affection) and unpleasant (anxiety, hurt, sadness, loneliness and hopelessness). Note that the pleasant emotions vary more strongly on the level of activation (euphoria v. contentment) than the unpleasant emotions (anxiety v. sadness).

The following analyses focus on the incidence of the ten emotions during the middle 14 days of the study (from Sunday to Saturday). During the initial session, respondents

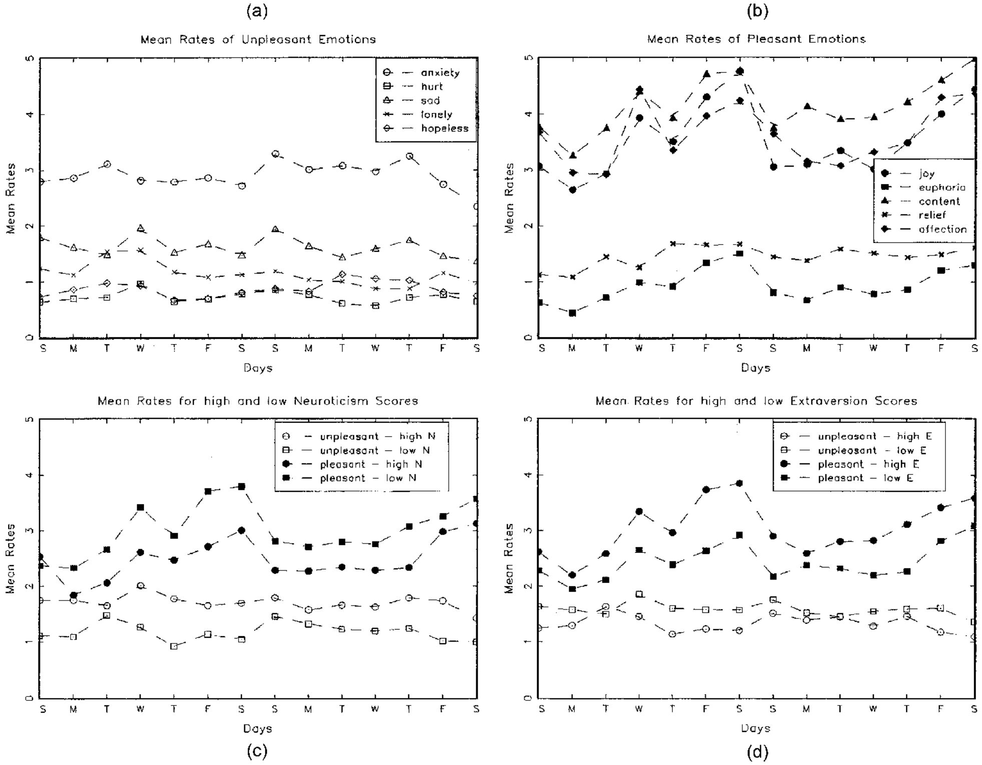

Figure 1. Main ratings of (a) unpleasant and (b) pleasant emotions, and their relationship to (c) neuroticism and (d) extraversion scores.

also answered a series of questions that included the two personality factors mentioned above. Plots of the observed daily average frequencies of emotional experiences are presented in the four panels of Fig. 1. Figures 1(a) and 1(b) display the unpleasant and pleasant emotions, respectively. We note that the ordering of the mean rates for the different emotions is fairly stable across the 14 days. As expected, the mean rates appear to vary systematically as a function of the days of the week. For example, on Mondays respondents report a lower rate of pleasant emotions than on other days. The unpleasant emotions seem to be more stable than the pleasant emotions, which may be related to the fact that the pleasant emotions differ more in their activation levels.

To illustrate relationships between the personality factors and the pleasant and unpleasant emotions, panel members were assigned to 'low' and 'high' categories on the basis of a median split for each of the two personality scores, neuroticism and extraversion. As can be seen from Figs. 1(c) and 1(d), the observed mean rates for unpleasant emotions are strongly related to both the neuroticism and extraversion scores: respondents with high neuroticism scores experience more frequently unpleasant and less frequently pleasant emotions than respondents with low neuroticism scores. In contrast, respondents with high extraversion scores experience more frequently pleasant and less frequently unpleasant emotions than respondents with low extraversion scores. From the plots, effects of the personality factors appear to be remarkably stable over time. Although these plots indicate that extraversion and neuroticism are related to emotional experiences, their separate effects need to be interpreted with some care because the correlation between the personality scores is − 0.37 in this sample.[1]

4.2. Analyses and results

The purpose of the following analyses was to determine whether the underlying dependency structure of the daily count data is consistent with a two-dimensional factor-analytic model. To address this question, we treated the 14 daily data sets as replications and fitted the one-, two- and three-dimensional factor models (see (2)) for each day separately. The results of these analyses are reported in the four panels of Fig. 2. Figure 2(a) contains a plot of the BIC statistics of the three models. Although there is little doubt that the one-dimensional factor model is inappropriate for this data set, the differences in fit between the two- and three-factor models are less pronounced. The two-dimensional factor model yielded a lower BIC statistic than the three-dimensional factor model for 10 of the 14 days, but the improvement in the BIC provided by the three-factor model in the remaining four cases was small. Moreover, item-specific overdispersion effects did not prove to be important. Only for one of the 14 days did the two-factor negative binomial model in (7) yield a lower BIC statistic than the two-factor Poisson model.

Detailed residual analyses of both models revealed that the two- and three-factor models differed little in the extent to which they fitted the mean and covariance structures of the daily data. The fit of the two-factor model is illustrated in the remaining three panels of Fig. 2. Figure 2(b) contains boxplots—with the 5th and 95th percentile as lower and upper limits—of the 45 Pearson's statistics obtained by comparing observed and expected frequencies for all pairs of the ten items. These fit statistics were computed by collapsing cells containing three or more reported emotions to obtain a 4×4 frequency table for each emotion pair. Although Pearson's test statistic for grouped data does not follow asymptotically a χ^2 distribution under the null hypothesis, an upper bound of the limiting distribution of the test statistic is provided in our case by a χ^2 distribution with $16 - 1$ degrees of freedom (Chernoff & Lehmann, 1954). We therefore include the 5th and 95th percentile of a χ^2 variate with 15 degrees of freedom as dotted lines in the plot. The boxplots show that the two-factor model yields a satisfactory fit of the item pairs for the majority of the 14 days. The two-factor model also provides a satisfactory fit of the marginal distributions of the emotion items. This is illustrated in Figs. 2(c) and 2(d), which contain the observed and predicted mean rates and the (log-transformed) variances. Because similar plots showed that the three-factor model did not yield an appreciably better fit for poorly fitted item pairs, we selected the two-factor model for subsequent analyses.

Figure 3 contains the values of the factor weights for the pleasant and unpleasant emotions and the two factors separately. The coefficients of the first factor are positive. We tend to interpret this factor as the activation dimension. For the pleasant emotions (Figs. 3(b) and 3(d)), the weights of 'euphoria' and 'contentment' are most separated, indicating that this part of the factor may be related to the activation level of the

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[1] The square-root transformed, daily sums of pleasant and unpleasant emotions were submitted to a multivariate analysis of variance. This analysis revealed significant linear and quadratic weekday effects for pleasant emotions and significant linear weekday effects for unpleasant emotions. Neuroticism proved to be a significant between-groups factor for unpleasant emotions. Extraversion did not have any significant between-subjects effects. These significance results need to be treated with caution because the normality assumption was violated for each of the 14 days for the unpleasant emotions and for 10 of the 14 days for the pleasant emotions.

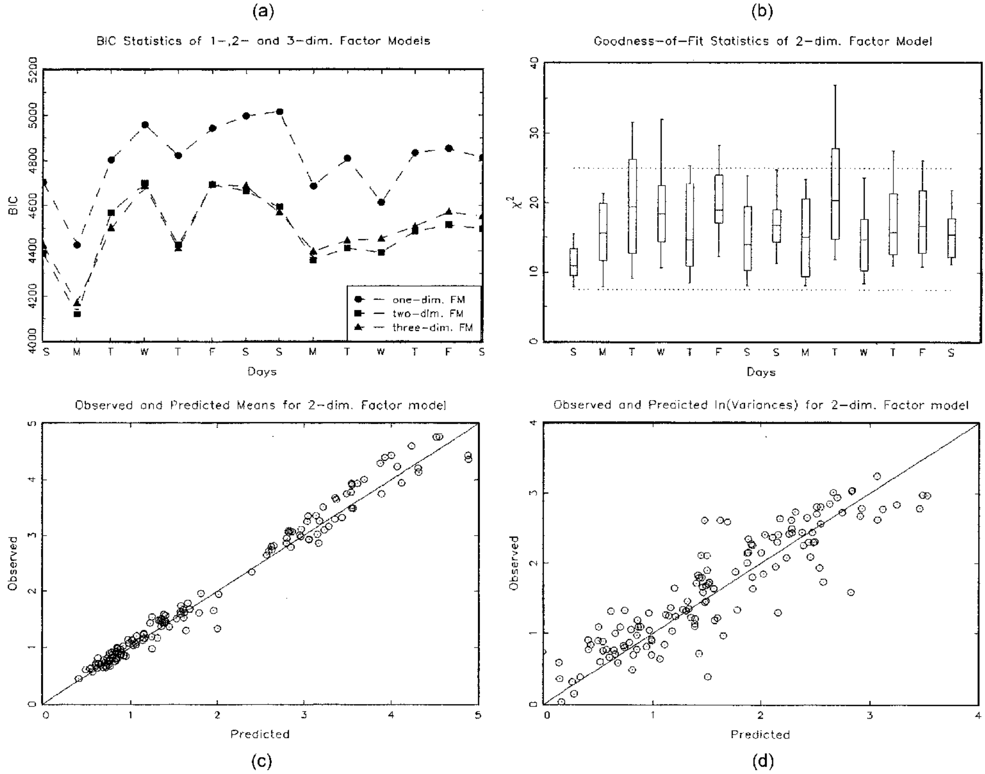

Figure 2. Fit statistics of Poisson factor models: (a) BIC statistics for one-, two- and three-dimensional factor models; (b) goodness-of-fit statistics for two-dimensional factor model; (c) observed and expected means for two-dimensional factor model; (d) observed and expected log variances for two-dimensional factor model.

emotions. A similar but much less pronounced relationship is observed for the unpleasant emotions (Figs. 3(a) and 3(c)). This result is in line with our previous observations that the unpleasant emotions were more stable (Fig. 1) and varied less in their activation levels. The orderings of the weights corresponding to the unpleasant emotions appear to be similar for both factors, which seem to indicate an interactive effect: the activation dimension seems to be reflected in the frequency of pleasant emotional experiences, but not in unpleasant ones. The weights of the second factor (Figs. 3(c) and 3(d)) clearly separate the pleasant from he unpleasant emotions, with 'euphoria' as the most pleasant and 'hopelessness' as the most unpleasant effect. We therefore interpret the second factor as the pleasantness factor. Note that this factor structure creates a negative covariance between pleasant and unpleasant emotional experiences.

To investigate the influence of personality characteristics on the structure of emotions, we included the two personality factors neuroticism and extraversion as predictors for the factor scores according to (4). For each of the 14 days, this investigation showed that both factors are highly associated with the extraversion and neuroticism scores. The average G^2 statistic obtained when setting $\mathbf{B} = \mathbf{0}$ is equal to 18.2 ($df = 4$). For two days (the first Monday, $G^2 = 8.1$, and second Saturday,

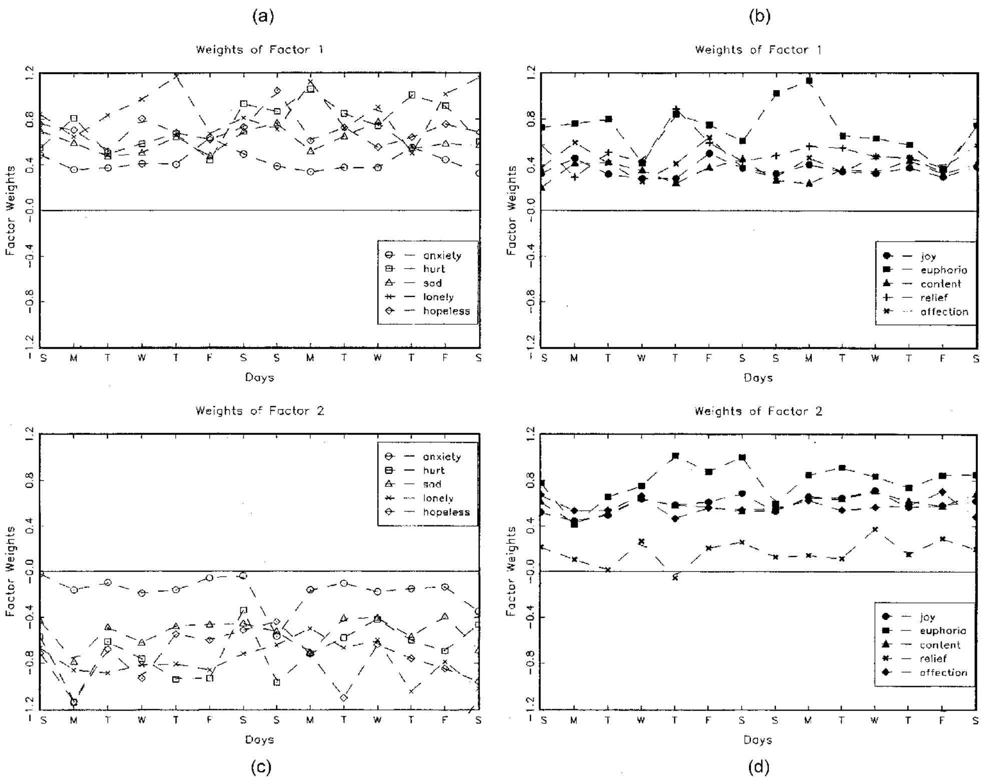

Figure 3. Weights of factor 1 for (a) unpleasant and (b) pleasant emotions, and of factor 2 for (c) unpleasant and (d) pleasant emotions.

$G^2 = 9.6$), the G^2 statistics do not exceed the nominal .01 level. The effects of the personality variables are illustrated in Fig. 4, which display the emotion-specific regression weights ((γ_j^*) in (4)) for each of the 14 days and the two personality factors. Figures 4(a) and 4(b) show that neuroticism distinguishes very clearly between the pleasant and unpleasant emotions. Respondents who score high on this personality factor experience more unpleasant and fewer pleasant emotions than respondents with low scores. In contrast, the extraversion dimension is more strongly related to the scores of the first factor (Figs. 4(c) and 4(d)). In particular, 'euphoria' has significant regression weights, indicating that respondents who score high on extraversion have a stronger tendency to experience pleasant emotions.

Overall, these results appear to be consistent with Yik *et al.*'s (1999) conjecture of a two-dimensional 'core affect' structure. The fit of the two-dimensional model was not always conclusive on any given day. However, the relative stability of the factor weights over time and the residual analyses suggest that little is gained by postulating a higher-dimensional model. Although one of the two factors was strongly related to the hypothesized pleasantness dimension, evidence for the hypothesized activation dimension was less strong. It is possible that this result was caused by the reduced variation of the unpleasant affect items on this dimension, or by an interactive effect of activation level and pleasantness. Further research is necessary to address this issue.

The investigation of the relationship between the factor structure and the two personality factors demonstrated that neuroticism is strongly related to the experience

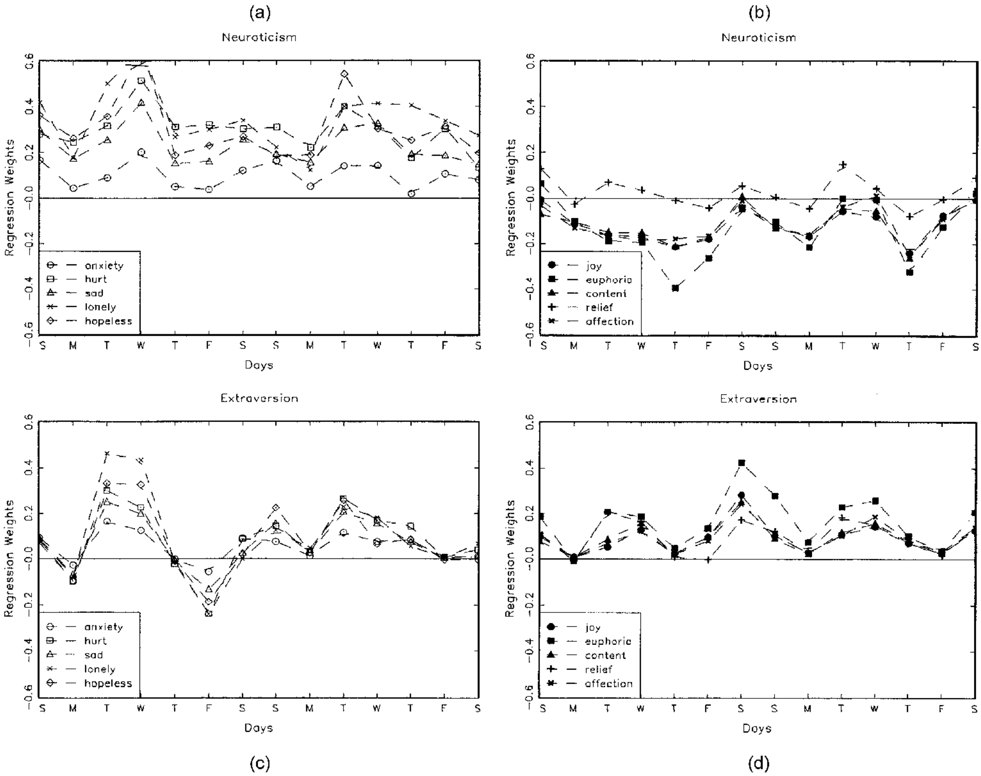

Figure 4. Factor score regression weights for neuroticism and (a) unpleasant and (b) pleasant emotions, and for extraversion and (c) unpleasant and (d) pleasant emotions.

of pleasant and unpleasant emotions. Moreover, our results indicate that extraversion is related to the activation levels of pleasant emotions. We found strong positive dependencies between extraversion scores and the incidence of 'euphoria' experiences. Because the extraversion scale assesses pleasurable arousal, it is possible that the 'semantic overlap' (Watson & Clark, 1992) of this concept with 'euphoria' produced an inflated estimate of the relationship in question. In a follow-up study it would be useful to include additional items with a wider range of unpleasant activation levels to distinguish better whether extraversion is more closely related to the experience of pleasant emotions or to the activation levels of both pleasant and unpleasant emotions.

The separate analyses of the daily self-report data revealed that the fluctuation of the factor and regression weights is appreciable on a day-to-day basis. By randomly selecting one of the days for the data analysis, one could have arrived at the conclusion that three as opposed to two factors are necessary in modelling the dependency structure of the counts. Clearly, this data set provided a useful perspective on the variability of the two-dimensional affect structure because the effects of emotional episodes and core affects cannot be separated on the basis of self-report data. As a next step, we plan to study the temporal dependencies among the emotions. Such an investigation would go beyond the Poisson factor models presented here because it would be necessary to take into account the autoregressive structure of the counts (Böckenholt, 1999). However, we expect that a dynamic version of the Poisson factor model may be well suited to studying the extent to which emotions affect each other beyond the day of their occurrence and whether 'carry-over' effects are moderated by personality factors.

5. Conclusion

The mixed-effects Poisson factor model discussed in this paper provides an alternative method for the analysis of dependency structures in multivariate count data. As a compromise between the J-dimensional random effects structure of the lognormal model and the constrained one-dimensional random-effects structure of the negative multinomial model, the Poisson factor model can be a solution to the abundance-of-parameters problem of the first and lack-of-fit problem of the second model. Clearly, the Poisson factor model is most appropriate when the covariation in the Poisson intensity parameters can be conceptualized as a result of a few latent, continuous variables. In this case, it provides a parsimonious, tractable and easy-to-interpret summary of the data. Moreover, the incorporation of covariates facilitates rigorous tests of hypotheses about the random-effects structure. The availability of estimation procedures and of goodness-of-fit tests adds to the usefulness of the mixed-effects Poisson factor model for exploring the structure of multivariate count data.

Acknowledgements

This research was partially supported by NSF grant SBR-9730197. The authors are grateful to Ulrich Schimmack for providing the data set used in Section 4.

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Received 27 March 2000; revised version received 6 June 2002