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Optimal Bundling and Pricing under a Monopoly: Contrasting Complements and Substitutes from Independently Valued Products*

We develop an analytical model of contingent valuations and address two questions of import to a monopolist: (i) should a given pair of complements or substitutes be sold separately (pure components), together (pure bundling), or both (mixed bundling), and at what prices? (ii) How do optimal bundling and pricing strategies for complements and substitutes differ from those for independently valued products? We find that the combination of marginal cost levels and the degree of complementarity or substitutability determines which of the three bundling strategies is optimal. Complements and substitutes should typically be priced higher than independently valued products.

I. Introduction

Marketing literature has witnessed a recent spurt in articles devoted to the study of bundling.[1] Manifestations of bundling include vacation packages, magazine subscriptions, and software bundles. Much of the academic work in marketing on bundling is rooted in seminal theories developed by economists (e.g., Adams and Yellen 1976). These studies rely on the reservation price paradigm and demonstrate the power of bundling as a price discrimination device.

Bundling articles commonly assume that a consumer's reservation price for the bundle is equal to the sum of his or her separate reservation prices for the component products (e.g., Adams and Yellen 1976,

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1. See, e.g., Hanson and Martin 1990; Venkatesh and Mahajan 1993; Yadav 1994; Ansari, Siddarth, and Weinberg 1996; Bakos and Brynjolfsson 1999. Bundling is the strategy of marketing products as specially priced packages (cf. Guiltinan 1987). Three alternative strategies are available to a seller: (i) pure components, in which the component products are sold separately; (ii) pure bundling, in which only the bundle is sold; and (iii) mixed bundling, in which the bundle and components are sold.

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p. 477; Schmalensee 1984, p. S213; McAfee, McMillan, and Whinston 1989, p. 373). This is called the assumption of strict additivity (Guiltinan 1987, p. 76). Products that conform to this assumption are referred to as independently valued products.

By contrast, managers often seek to maximize payoffs from interrelated products. When products are complements, a consumer's reservation price for the bundle is superadditive in those for the component products. Guiltinan (1987) suggests that complementarity arises because of search economies (e.g., oil and filter changes at the same gas station), enhanced customer satisfaction (e.g., ski rental accompanied by a lessons package), and improved total image (e.g., offering lawn care and shrub care services). Alternatively, when the products are substitutes, a consumer's reservation price for the bundle would be subadditive in those for the components. This is likely when the products offer (some) overlapping benefits (e.g., "Coke" and "Pepsi") or when they compete for similar resources such as a consumer's time.

The objective of this study is to model such contingent valuations formally and to offer normative guidelines on optimal bundling strategies and pricing patterns under a monopoly. To this end, we present an analytical model of contingent valuations from the standpoint of a profit-maximizing monopolist and use it to address two questions: (i) should a given pair of products be sold separately (pure components), together (pure bundling), or both (mixed bundling), and at what prices? (ii) How do the optimal bundling strategies and pricing patterns for complements and substitutes differ from those for independently valued products under various marginal cost conditions?

Our effort is aided by a handful of extant bundling articles that have recognized the importance of considering contingent valuations in bundling decisions. The key contributions of these studies and the distinctive features of our study are summarized in table 1.

These studies have examined contingent valuations at a conceptual level (e.g., Guiltinan 1987) and in a "narrow" setting (e.g., Anderson and Leruth [1993], who focus on perfect complements only). As we see it, ours is the first study formally to delineate the optimal bundling strategies and pricing patterns for interrelated products while recognizing that complementarity and substitutability are matters of degree and not a simple dichotomy. We demonstrate that optimal solutions for complements and substitutes are often quite different from those for independently valued products. We also show that marginal cost plays an important interactive role in determining the optimal strategy.[2]

We derive analytical results for pure components and pure bundling and use simulation analysis to compare them with mixed bundling. Results show that the optimal prices for complements or substitutes are mostly higher com-

2. We hasten to acknowledge that the extant studies in table 1 contain other results not within the scope of our study. For example, Bakos and Brynjolfsson (1999) provide several results on independently valued information goods. Along similar lines, Anderson and Leruth (1993) provide results under competition for perfect complements.

pared with those for independently valued products. Moreover, the three alternative strategies enjoy unique domains of optimality determined by a combination of marginal costs and the extent of contingent effects.

II. Model Development

Our seller is a monopolist who has stand-alone products 1 and 2 to offer in unit quantities. She may use pure components, pure bundling, or a mixed bundling strategy. The seller's objective is profit maximization in a static context. The market has M potential surplus-maximizing consumers, each of whom is willing to purchase one unit of each item at the most. The value of each product for a consumer in isolation of the other product is labeled as the "stand-alone reservation price." Several studies underscore the ratio property of the reservation price measure (e.g., Park and Srinivasan 1994). Consumers differ in their reservation prices for each product.

The notion of contingent valuation arises when a consumer considers the bundle. The consumer's reservation price for bundle "12" would be higher than (or less than) the sum of the stand-alone reservation prices for products 1 and 2 when the products are complements (or substitutes). For independently valued products, the reservation price for bundle 12 equals the sum of the stand-alone reservation prices for products 1 and 2. We assume that a consumer's reservation price for an explicit bundle (i.e., a bundle offered by the seller) is equal to that for an implicit bundle (i.e., a notional bundle that would arise if the consumer were to buy products 1 and 2 separately).

A consumer's degree of contingency (i.e., degree of complementarity or substitutability) is parameterized as Θ. For the pair of products 1 and 2, Θ for this consumer is defined as: $\Theta =$ (Reservation price for bundle 12 $-$ sum of stand-alone reservation prices for products 1 and 2)/(Sum of stand-alone reservation prices for products 1 and 2). Evidently, Θ is specific to a product pair. By the above definition, Θ would be zero for two independently valued products, positive for complements, and negative for substitutes. Measuring Θ as a ratio is consistent with Weber's law that consumers judge price and value changes in proportional terms rather than as absolute increments (Monroe 1990, p. 57). This approach to assess contingency is consistent with the literature (e.g., Venkatesh and Mahajan 1997).

The contingent valuation that we examine and correlation in reservation prices are distinct notions. While our degree of contingency parameter Θ captures perceived value enhancement or reduction within each consumer, the correlation in reservation prices for two products (as used by Schmalensee 1984) shows how stand-alone reservation prices relate to each other across consumers.

For simplicity and analytical tractability, Θ for a given pair of products is assumed to be a constant across consumers. This is akin to McGuire and Staelin's (1983, p. 169) constant Θ parameter and Bakos and Brynjolfsson's (1999, p. 1,621) constant α parameter, which are both intended to capture

TABLE 1 Benchmarking the Current Study with Other Studies on Contingent Valuations

Reference	Scope and Key Findings	Key Differences from Our Study
Anderson and Leruth 1993	Competitive bundling strategies in a duopoly for two complements are examined. Mixed bundling is found to be better suited for a monopoly. In a duopoly, pure components pricing is the equilibrium strategy.	Anderson and Leruth's study focuses on perfect complements (e.g., video cassette recorder and videotape). We consider a broad range of product pairs ranging from perfect substitutes to perfect complements. Unlike Anderson and Leruth we do not force a saturated market assumption.
Bakos and Brynjolfsson 1999	Under independent valuation, pure bundling is optimal for a monopolist marketing a large number of information goods with zero marginal costs. A subsection is devoted to contingent effects of the type we use. An inequality of profits under pure bundling is presented here.	Bakos and Brynjolfsson look at contingent valuations in passing. Their only result here is an inequality of profits under pure bundling. We offer general results on a wide range of contingent valuations. Unlike Bakos and Brynjolfsson, we allow marginal costs to take positive values and offer additional insights.
Carbajo, de Meza, and Seidmann 1990	Assuming perfect positive correlation and independent demand, study shows pure bundling is optimal under imperfect competition.	While all of Carbajo et al.'s six propositions are based on independent demand assumption, all of our results focus on contingent valuations. Unlike Carbajo et al., we also look at mixed bundling. Also, in contrast to their article, we consider substitutes and complements.
Eppen, Hanson, and Martin 1991	Using anecdotal examples and conceptual arguments on contingent valuations, Eppen et al. offer the guideline: "Use pure bundling when components perform better together than separately."	Eppen et al.'s conceptual paper offers a rationale for analytical studies such as ours to model bundling of "contingent" products. Eppen et al.'s guideline implies that pure bundling is optimal for all complements. We show that this is true for some strong complements only.

Guiltinan 1987	Offers an excellent introduction to the practical application of bundling strategies. The notion of contingent valuation is very well articulated.	Guiltinan's ideas form an important basis of our analytical model. We build on his preliminary ideas to offer analytically supported results.
Lewbel 1985	The notion of contingent valuations is recognized. Two theorems on the effects of price changes on buyer decisions are presented. Two anecdotal examples are used to show that Adams and Yellen's (1976) results may be violated when contingent effects are present.	Lewbel supports the validity of the issue of contingent valuation that we address. With regard to optimal bundling strategies, Lewbel focuses on identifying violations of earlier generalizations rather than on proposing new generalizations. Our focus is on the latter.
Wilson, Weiss, and John 1990	The relative attractiveness of pure components and pure bundling is examined for modules of industrial systems in a competitive market. Unbundling is preferable when new systems that can be put together are superior.	While Wilson et al. focus on perfect complements only, we consider the entire range from perfect substitutes to perfect complements. Whereas Wilson et al. consider pure components and pure bundling only, we also consider mixed bundling.
This study	Strategies for complements, substitutes, and independently valued products are contrasted. Each of the three strategies has its unique domain of optimality defined by the extent of interrelatedness among products and the level of marginal costs. Optimal prices under pure components and pure bundling are typically higher for substitutes and complements compared with independently valued products.	Not applicable

value dependency. Some implications of heterogeneity in Θ are presented in the discussion section.

Analytical Underpinnings

The stand-alone reservation price of consumer r for product i is denoted by R_i^r (for $i = 1, 2$). This consumer's reservation price for the bundle comprising one unit each of products 1 and 2 is denoted by R_{12}^r. The degree of contingency for products 1 and 2 is given by Θ^r and defined as

$$\Theta^r = \frac{R_{12}^r - (R_1^r + R_2^r)}{R_1^r + R_2^r}. \quad (1)$$

As we assume that Θ^r is a constant for a given product pair 1 and 2 and equal to Θ, we have:

$$R_{12}^r = (1 + \Theta)(R_1^r + R_2^r). \quad (2)$$

The seller's unit price of product i ($i = 1, 2$) is P_i and that of a bundle is P_{12}. The seller's marginal cost of each product is c_i and that of a bundle is assumed to be $c_1 + c_2$. Fixed costs are set to zero without loss of generality. Importantly, P_i, under pure components, is allowed to be different from that under mixed bundling. Similarly, P_{12} is not constrained to be the same under pure and mixed bundling. However, additional suffixes are avoided to keep the notation simple.

Under pure components, a consumer has the option to purchase product 1, product 2, both, or neither, depending on which option yields the highest surplus. The consideration set is $\{1, 2, (1, 2), \phi\}$, where ϕ denotes no purchase. The corresponding set of surpluses is $\{(R_1^r - P_1), (R_2^r - P_2), [(1 + \Theta)(R_1^r + R_2^r) - P_1 - P_2], 0\}$. When the consumer's best purchase option is only as attractive as the no-purchase option, the consumer is assumed to purchase the product.

Under pure bundling, the consideration set is $\{12, \phi\}$, wherein 12 represents the bundle of products 1 and 2. When products 1 and 2 are substitutes, a fraction of the consumers may purchase the bundle with the intent of using product 1 alone or product 2 alone. The set of surpluses for a randomly drawn consumer is $\{(R_1^r - P_{12}), (R_2^r - P_{12}), [(1 + \Theta)(R_1^r + R_2^r) - P_{12}], 0\}$.

Under mixed bundling, a consumer's consideration set would be $\{1, 2, 12, \phi\}$. Like Schmalensee (1984, p. S224), we constrain the bundle price to be weakly subadditive in the component prices, that is, $P_{12} \leq (P_1 + P_2)$. This ensures survivability of the bundle and does not let mixed bundling degenerate into pure components, which is a case we examine separately. So, purchasing the bundle is generally more attractive for a consumer than purchasing products 1 and 2 in stand-alone form. The set of surpluses is $\{(R_1^r - P_1), (R_2^r - P_2), [(1 + \Theta)(R_1^r + R_2^r) - P_{12}], 0\}$.

The distribution of stand-alone reservation prices for products 1 and 2 across consumers is represented by the bivariate probability density function $f(x, y)$. Models of the seller's profits under alternative scenarios are explained next.

Bundling Strategies and Profits for Two Complements or Substitutes

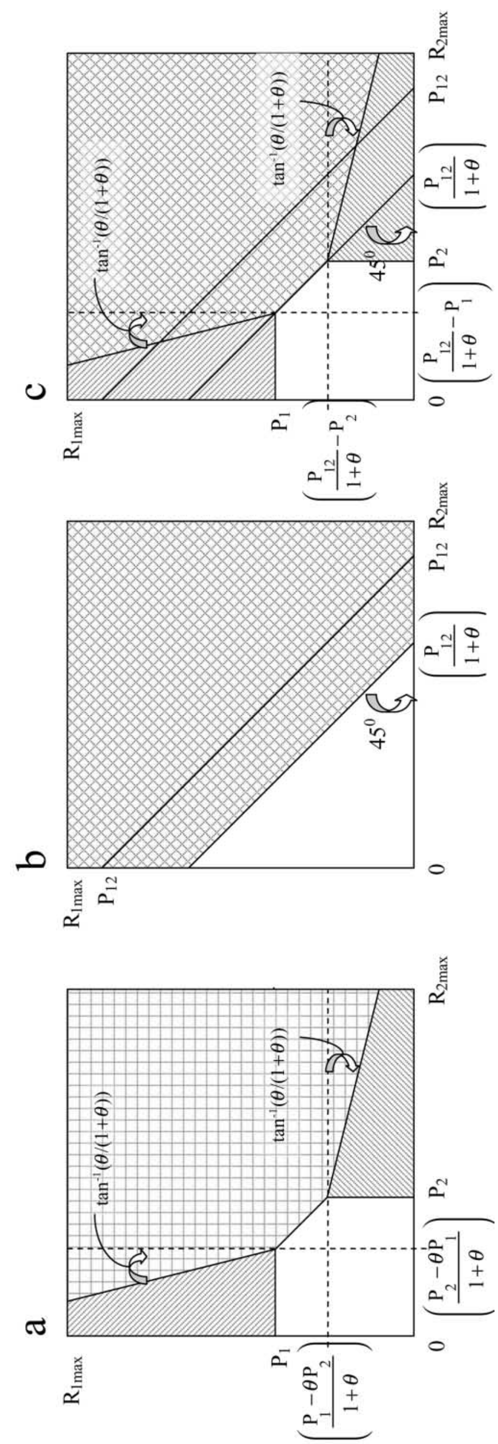
Consumers' willingness to pay for the bundle of complements 1 and 2 is higher compared with their willingness to pay for a bundle of two independently valued products with the same stand-alone reservation prices. Thus, for a given price, a higher proportion of the market buys the complement pair compared with the corresponding pair of independently valued products under each bundling strategy. The expected penetration pattern is schematically represented in figures 1*a*–*c*. The vertical and horizontal axes represent the stand-alone reservation prices of products 1 and 2, respectively.

The opposite happens for substitutes. The subadditive nature of the reservation prices for the two products causes sizable segments of the market to purchase only one of the two products. Figures 1*d*–*f* contain the schematic representation of this case. Although the optimal prices for complements and substitutes are allowed to differ from each other, no suffixes are added to P_1, P_2, and P_{12} for notational simplicity.

Figure 1 provides some insights into why the conventional guidelines on optimal bundling and pricing for independent goods may not apply for complements and substitutes. Conventional wisdom (cf. Schmalensee 1984) suggests that pure bundling works by reducing buyer heterogeneity and focusing on consumers who value both products; pure components is attractive in tapping consumers who are willing to pay a high price for one product but not for the other; and mixed bundling typically emerges as the optimal strategy, as it blends the advantages of pure components and pure bundling. As a contrasting example, figure 1*c* suggests that when most consumers prefer to buy two complements, the components under mixed bundling have a tighter and less typical role to perform. Again, as shown in figure 1*f*, when fewer consumers have a desire to purchase two substitutes, the appeal of the bundle is diminished and the traditional approach of targeting high-priced components for consumers at the fringes seems narrow. We will formally examine these contrasts after the rest of the model is set up.

We use a common set of equations (see below) to represent unit sales and profits from complements and substitutes. Figures 1*a* and 1*d* capture the penetration under pure components for complements and substitutes, respectively. Unit sales of either product under pure components is given by

$$\text{Unit sales of } i = A_i = M \times \Pr\left[\left\{\begin{matrix}(R_i \geq P_i) \cap \\ [(R_i - P_i) > (R_j - P_j)]\end{matrix}\right\} \cup \left(\begin{matrix}[(1+\Theta)(R_1 + R_2) \geq (P_1 + P_2)] \cap \\ \{[(1+\Theta)(R_1 + R_2) - (P_1 + P_2)] \geq (R_j - P_j)\}\end{matrix}\right)\right], \tag{3}$$


a
b
c
R1max
P1
P12
P2
R2max
0
45⁰
tan⁻¹(θ/(1+θ))
(P1 − θP2)/(1+θ)
(P2 − θP1)/(1+θ)
P12/(1+θ)
P12/(1+θ) − P1
P12/(1+θ) − P2

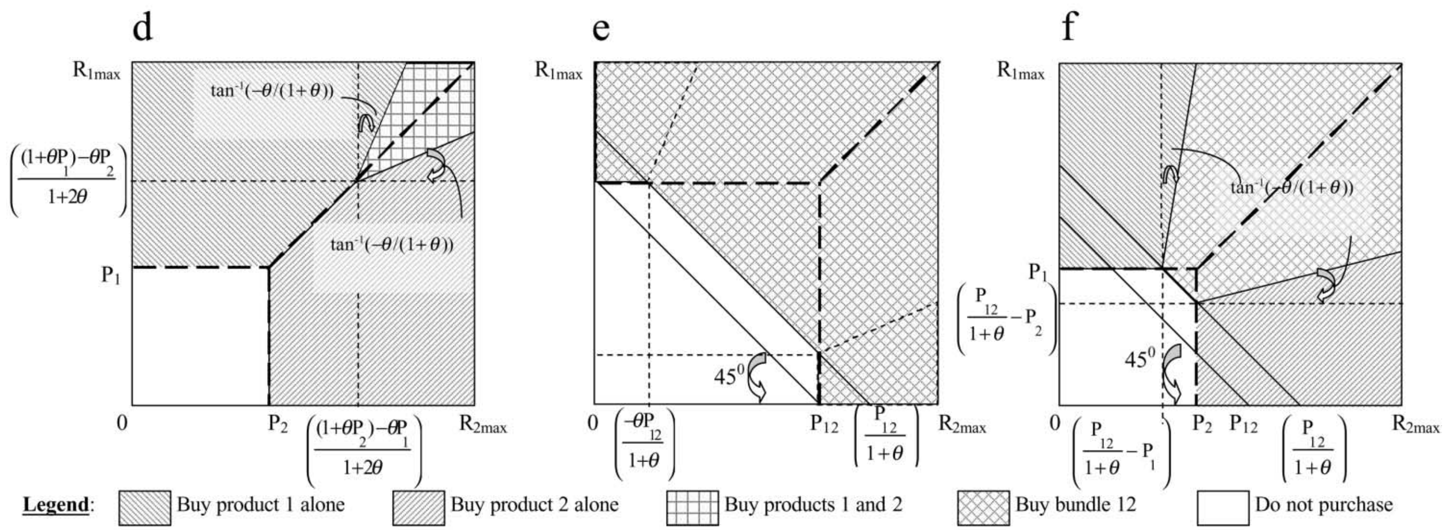


FIG. 1.—Bundling with two complements or substitutes. Pricing and penetration with two complements: *a*, Pure components. *b*, Pure bundling. *c*, Mixed bundling. Pricing and penetration with two substitutes: *d*, Pure components. *e*, Pure bundling. *f*, Mixed bundling.

for $i, j = 1, 2$ and $i \neq j$, where $\Pr(\cdot)$ ensures that consumers who purchase i derive their highest, nonnegative surplus from i alone or from an implicit bundle that they can form of products 1 and 2. Thus, $\Pr(\cdot)$ ensures that both the participation constraint and incentive compatibility constraint are met. The difference between complements and substitutes is in the magnitude of $\Pr(\cdot)$. When products 1 and 2 are independently valued, unit sales of i would be $M \times \Pr(R_i \geq P_i)$.

The seller sets the optimal price of products 1 and 2 using the objective function

$$\max_{P_1, P_2} \prod = \sum_{i=1}^{2} (P_i - c_i) \times A_i, \tag{4}$$

where A_i is as defined in equation (3) and $\prod$ denotes profits.

Penetration under pure bundling is represented in figures 1*b* and 1*e* for complements and substitutes, respectively, and is given by:

Unit sales of bundle $12 = B$

$$= M \times \Pr\{[(1 + \Theta)(R_1 + R_2) \geq P_{12}] \cup (R_1 \geq P_{12}) \cup (R_2 \geq P_{12})\}. \tag{5}$$

When Θ is zero, the expression for unit sales will simplify to $M \times \Pr[(R_1 + R_2) \geq P_{12}]$.

The optimal price P_{12} is determined using the objective function for profit maximization:

$$\max_{P_{12}} = [P_{12} - (c_1 + c_2)] \times B. \tag{6}$$

Penetration under mixed bundling is represented in figures 1*c* and 1*f* and is given by:

$$\text{Unit sales of } i = C_i$$

$$= M \times \Pr\begin{pmatrix} (R_i \geq P_i) \cap \{(R_i - P_i) > [(1 + \Theta)(R_1 + R_2) - P_{12}]\} \\ \cap[(R_i - P_i) > (R_j - P_j)] \end{pmatrix} \tag{7a}$$

for $i, j = 1, 2$ and $i \neq j$.

$$\text{Unit sales of bundle } 12 = D$$

$$= M \times \Pr\begin{pmatrix} [(1 + \Theta)(R_1 + R_2) \geq P_{12}] \cap \{[(1 + \Theta)(R_1 + R_2) - P_{12}] \geq (R_1 - P_1)\} \\ \cap\{[(1 + \Theta)(R_1 + R_2) - P_{12}] \geq (R_2 - P_2)\} \end{pmatrix}. \tag{7b}$$

Notice again, in equations (7a) and (7b), that $\Pr(\cdot)$ takes care of participation and incentive compatibility constraints. Mixed bundling underscores the potential to enhance profits by selling more bundles of complements (see fig. 1*c*). However, bundle sales of substitutes would be lower (see fig. 1*f*).

For independently valued products under mixed bundling, unit sales of product *i* and bundle 12 are inferred from equations (7a) and (7b) by setting Θ to zero. The optimal prices of the bundle (P_{12}) and the component products 1 and 2 (P_1, P_2) are based on the following objective function to maximize profits:

$$\max_{P_1,P_2,P_{12}} \prod = \sum_{i=1}^{2} (P_i - c_i) \times C_i + [P_{12} - (c_1 + c_2)] \times D, \tag{8}$$

where C_i and D are defined in equations (7a) and (7b).

The thick broken lines in figures 1*d*–1*f* capture the penetration under the three strategies when the degree of substitutability reaches a certain threshold. Both at and higher than this level of substitutability, consumers restrict their purchase or consumption to only one of the two products. This threshold value of Θ is -0.5 under pure and mixed bundling. Under pure components, the threshold is $\Theta \approx -0.2$, or, specifically, $\Theta = (c + \sqrt{c^2 + 3a^2} - 3a)/6a$.

III. Analytical Results for Pure Components and Pure Bundling

We now address the decision questions raised up front for pure components and pure bundling using analytically derived results. Mixed bundling does not lend itself to closed-form solutions. Therefore, in Section IV, we rely on simulation to compare mixed bundling with the pure strategies and answer the decision questions more generally.

We make two additional assumptions. We assume that the bivariate distribution of stand-alone reservation prices follows the uniform density $f(x, y) = 1/a^2$ for $0 \leq R_i \leq a$, $i = 1, 2$. The reasons for using the uniform density are twofold. First, the uniform distribution and its close cousin—the linear demand function—have been employed widely in normative models in bundling (e.g., Carbajo, de Meza, and Seidmann 1990, p. 287; Seidmann 1991, p. 492; Matutes and Regibeau 1992, p. 39) and in other areas (e.g., McGuire and Staelin 1983, p. 168). Second, this form is analytically quite tractable. We also assume, for purposes of notational parsimony, that the unit variable costs of product 1 and 2 are equal; that is, $c_1 = c_2 = c$. Given these two assumptions, it can be shown that the optimal, equilibrium prices P_1 and P_2 of products 1 and 2 are equal.[3] These assumptions also let us contrast the optimal prices and strategies for complements and substitutes relative to independently valued goods purely as functions of Θ and c/a.

3. The proof is available from the authors. The intuition is straightforward. Notice that with the assumptions of a symmetric bivariate distribution in reservation prices and equality in marginal costs, the two products that we model have no inherent asymmetry on the demand and supply sides.

Closed-Form Solutions under Pure Components and Pure Bundling

The optimal prices P_i of products i under pure components and P_{12} of bundle 12 under pure bundling may be expressed in closed form in terms of demand-side variables Θ and a and supply-side variable c. The resulting solutions are listed below (the interested reader may contact the first author for analytical derivations of optimal prices listed below and for proofs of results 1–3). While optimal prices for independently valued goods may be inferred from earlier studies (e.g., Schmalensee 1984), those for complements and substitutes have been derived specifically for the current study using our proposed model.

Optimal prices for independently valued products ($\theta = 0$). Pure components:

$$P_1 = P_2 = \frac{c + a}{2}.$$

Pure bundling:

$$P_{12} = \frac{2c + \sqrt{(2c)^2 + 6a^2}}{3} \text{ for } c \leq \frac{a}{4};$$

$$= \frac{2a + 4c}{3} \text{ for } c > \frac{a}{4}.$$

Optimal prices for complements ($\theta > 0$). Pure components:

$$P_1 = P_2 = \frac{c}{3} + \frac{2a(1+\theta)}{3\theta(\theta-3)}$$

$$- \frac{\sqrt{[4a(1+\theta) + 2c\theta(\theta-3)]^2 - \{12\theta(\theta-3)(1+\theta)[a^2(3\theta+2) + 2ac]\}}}{6\theta(\theta-3)}$$

for $0 < \theta < P_1/a;$

$$= \frac{c}{3} + \frac{\sqrt{p[pc^2 + 6a^2\theta(1+\theta)^2]}}{3p},$$

where

$$p = \theta^2(3-\theta) + (1+\theta) \text{ for } \theta > P_1/a.$$

Pure bundling:

$$P_{12} = \frac{2c + \sqrt{(2c)^2 + 6a^2(1+\theta)^2}}{3} \text{ for } c < \frac{a(1+\theta)}{4}$$

$$= \frac{2a(1+\theta) + 4c}{3} \text{ for } c \geq \frac{a(1+\theta)}{4}.$$

Optimal prices for substitutes ($\theta < 0$). Pure components:

$$P_1 = P_2 = \frac{-2(m - lc) - \sqrt{4(m - lc)^2 - 12l(n - mc)}}{6l}$$

$$\text{for } \frac{c + \sqrt{c^2 + 3a^2} - 3a}{6a} < \theta < 0,$$

where

$$l = -\theta(1 + 2\theta)(3 + 2\theta);\, m = -2a(1 + 2\theta)^2$$

$$\text{and } n = a^2(1 + 2\theta)^2(2 + 3\theta);$$

$$P_1 = P_2 = \frac{c + \sqrt{c^2 + 3a^2}}{3} \text{ for } \theta \leq \frac{c + \sqrt{c^2 + 3a^2} - 3a}{6a}.$$

Pure bundling:

$$P_{12} = \frac{2c + \sqrt{(2c)^2 - \frac{6a^2(1+\theta)^2}{2\theta^2 - 1}}}{3} \text{ for } -0.5 < \theta < 0$$

$$\text{and } c \leq \frac{a}{4}\left(\frac{8\theta^2 + 4\theta - 1}{2\theta^2 - 1}\right);$$

$$= \frac{2a(1 + \theta) + 4c}{3} \text{ for } -0.5 < \theta < 0$$

$$\text{and } c > \frac{a}{4}\left(\frac{8\theta^2 + 4\theta - 1}{2\theta^2 - 1}\right)$$

$$= \frac{2c + \sqrt{(2c)^2 + 3a^2}}{3} \text{ for } \theta \leq -0.5.$$

Guidelines on Optimal Pricing

Pricing patterns under the pure components strategy.

Result 1. Under pure components,

a) optimal prices of complements are monotonically increasing in Θ.
b) optimal prices of all but the very weak substitutes are higher than those of independently valued products.

Comment. The patterns are illustrated in figure 2 for two cost levels. Result 1 and the figure underscore that optimal prices under pure components are different from, and typically higher than, those for independently valued products.

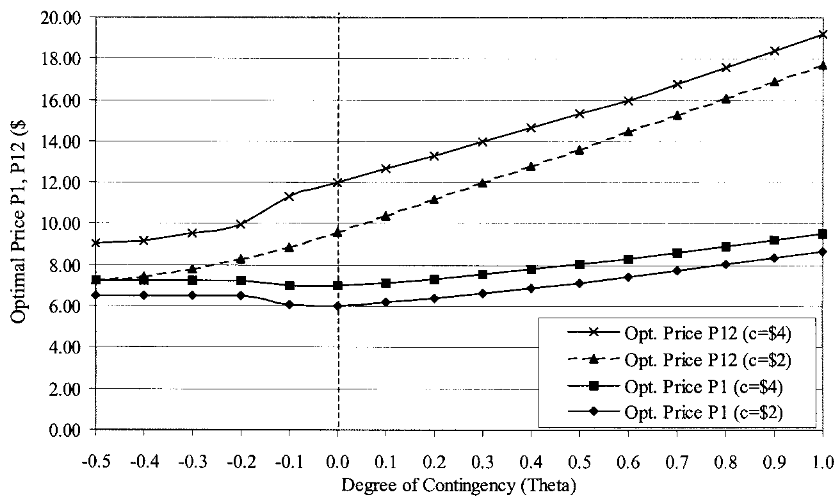


FIG. 2.—Optimal prices under pure components and pure bundling (analytical results). P_1, P_{12} = optimal prices of the component product and bundle, respectively; c = marginal cost of each component product.

The rationales differ for complements and substitutes. As reservation prices for a pair of complements ($\Theta > 0$) exceed those for independently valued goods ($\Theta = 0$), the seller gains by charging higher prices even while stimulating more consumers to buy both products. With substitutes ($\Theta < 0$), the lower the Θ the greater the consumers' reluctance to buy a pair of products (shown graphically in fig. 1*d*). Here, instead of offering a subsidy to entice a shrinking segment of buyers of both products, the seller is better off charging a higher price to buyers of a single product. The exception is for very weak substitutes ($-0.11 < \Theta < 0$ for $c/a = 1/5$ and $-0.06 < \Theta < 0$ for $c/a = 2/5$, where a is the market's maximum stand-alone reservation price).[4] Here, the products are "competing" only slightly with each other in the consumer's perception. Offering a shallow subsidy (relative to the prices for independently valued goods) to encourage joint purchase is profit maximizing. (Notice that this narrow band of weak substitutes becomes narrower as c/a increases.)

Pricing under the pure bundling strategy.

Result 2. Under pure bundling, optimal prices of complements and substitutes are monotonically increasing in Θ.

Comment. The stronger the complementarity (signified by a higher, positive Θ), the greater the consumers' reservation prices for the bundle. Such

4. For the uniform heterogeneity that we have assumed, the market's maximum stand-alone reservation price and the range in stand-alone reservation prices are both a for either product. However, in the context of the c/a ratio, it is useful to think of a as the maximum stand-alone reservation price. Doing so makes it very obvious that when $c \rightarrow a$, the maximum stand-alone reservation price, the product by itself becomes incapable of yielding any profit to the seller.

enhancement in reservation prices associated with increasing Θ is conducive to charging higher bundle prices. The converse is true for substitutes (see fig. 2 for illustrations at two cost levels). There is an added difference. The stronger the substitutability, the greater the proportion of buyers who intend to use only one of the two products in the bundle and discard the other. This forces the seller to lower the bundle price. When Θ drops to -0.5, each buyer of the bundle intends to use one product only. The declining bundle price reaches its lowest point.

Even while alerting the seller to leverage complementarity and charge higher prices for increasing Θ, the result exposes the difficulty of managing substitutes via a pure bundling strategy. In the context of independently valued goods, Adams and Yellen (1976) make a telling point that a monopolist's decision to bundle can lead to oversupply or undersupply of particular goods. This problem becomes particularly severe with substitutes, especially when variable costs are sizable (see fig. 1*e* for a graphic representation). The consequent low value (resulting from substitutability) and high price of the bundle (resulting from sizable marginal costs) causes many consumers to forgo purchase entirely, while certain other consumers buy the bundle and discard one of the products. Pure bundling ends up being less attractive for substitutes. These issues are examined more completely below.

Relative Attractiveness of Pure Components and Pure Bundling

Based on the analytical solutions for prices, the exact profit from either strategy can be obtained for any combination of Θ and c/a. This has yielded the phase diagram in figure 3*a*.

We rely on figure 3*a* to clarify the relative attractiveness of pure components and pure bundling. Here, we distinguish between "low," "moderate," and "high" levels of the c/a ratio.

Result 3.

a) For low levels of c/a, (i) pure bundling is more profitable than pure components for weak substitutes, independently valued products, and complements; (ii) pure components is profitable for strong substitutes.

b) For moderate levels of c/a, (i) pure components is more profitable for all cases except the very strong complements; (ii) pure components and pure bundling are equally profitable for very strong complements.

c) For high levels of c/a, (i) pure components is more profitable for substitutes, independently valued products, and weak complements; (ii) pure components and pure bundling are equally attractive for moderate-to-strong complements.

Comment. Consider result 3*a*. The low levels of relative marginal costs let the seller price the bundle low enough to make the bundle appealing even within segments of consumers who would normally buy just one product. Bundling is an "inclusive" mechanism here. This explains part i. However,

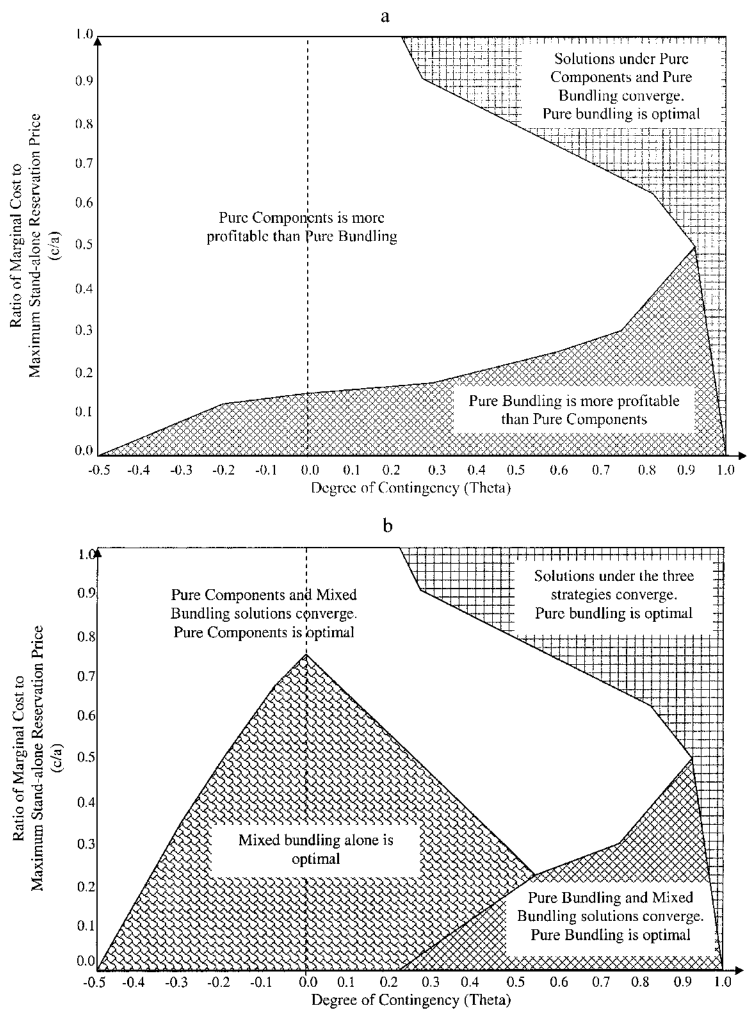


FIG. 3.—Optimality of alternative bundling strategies. *a*, Phase diagram comparing pure components and pure bundling (analytical results). *b*, Phase diagram comparing all three alternative strategies (simulation results).

for strong substitutes (pt. ii), consumers' aversion for the bundle is so high that even with zero marginal costs, the seller finds pure components more attractive.

Under 3*b*, part i, the moderate levels of relative marginal costs push the bundle prices so high that consumers who seek both products are pretty much the only ones who find the bundle attractive. In contrast, pure components draws a sizable fraction of the segments wanting just a single product while also having a fair appeal for consumers seeking both products. The exception is for strong complements (pt. ii). The strong complementarity makes the bundle much more attractive than a single product. Pure components becomes de facto the pure bundling strategy in that all buyers of one product also buy the other and form their own implicit bundle.

For high levels of relative marginal costs, under 3*c*, part i, the rationale under 3*b*, part i, becomes even more pertinent. A different logic applies for result 3*c*, part ii. The combination of high relative costs and moderate-to-strong complementarity makes it optimal for the seller to ignore consumers who want just one product. So the two strategies converge to pure bundling. At the optimal prices, all buyers purchase both products under pure components.

The result shows that the seller's pick between pure components and pure bundling depends on the combination or interaction of the degree of contingency (Θ) and marginal costs. What happens when marginal cost c is zero? This requires taking result 3*a* to the limit:

COROLLARY 1. When $c = 0$, pure bundling is weakly more profitable than pure components.

Let us now examine the effectiveness of the pure strategies relative to mixed bundling.

IV. Generalized Results through Simulation

Under mixed bundling, we have to consider both component and bundle prices simultaneously. So a closed-form solution is not feasible (cf. Wilson 1993), and we resort to simulation. This problem did not arise for pure components and pure bundling because in our setup the screening mechanism is unidimensional. Numerical analysis has been used earlier to study bundling (e.g., Schmalensee 1984). Our simulations use a uniform distribution of reservation prices, $f(x, y) = 1/(10^2)$, for $0 \leq R_i \leq \$10$, $i = 1, 2$, and unit costs ($c_1 = c_2 = c$) ranging from zero to \$10 in unit increments. A population of 90,000 "synthetic consumers" is generated. For a given vector of prices, each consumer makes the purchase decision that maximizes the consumer surpluses defined in Section II. These individual decisions for each of the 90,000 synthetic consumers define the aggregate demand for the components and bundle.

The monopolist's optimal prices under pure components, pure bundling, and mixed bundling strategies are obtained numerically. In view of the potential discontinuities in the profit function, caused by the simulation of demand, we use a polytope optimization algorithm for nonsmooth objective functions (Gill, Murray, and Wright 1981). The analytical and simulation approaches yield the same results for pure components and pure bundling, while the simulation approach allows us to explore situations for which closed-form solutions are not tractable.

Optimality of Alternative Strategies

The optimal bundling strategy for a combination of degree of contingency Θ and the level of marginal cost relative to the maximum willingness to pay, that is, c/a, is presented in figure 3*b*.

We rely on this phase diagram to draw the following results:

Result 4.

a) Under low levels of c/a, (i) pure components is optimal for strong substitutes; (ii) mixed bundling is optimal for moderate-to-weak substitutes, independently valued products, and weak complements; (ii) pure bundling is optimal for moderate-to-strong complements.

b) Under moderate levels of c/a, (i) pure components is optimal for moderate-to-strong substitutes and moderate complements; (ii) mixed bundling is optimal for weak substitutes, independently valued products, and weak complements; (ii) pure bundling is optimal for strong complements.

c) Under high levels of c/a, (i) pure components is optimal for substitutes, independently valued products, and weak complements; (ii) pure bundling is optimal for moderate-to-strong complements.

Comment. Previous research underscores the power of mixed bundling as a price discrimination device for independently valued products (cf. Schmalensee 1984). Result 4*a* shows that under low levels of relative marginal costs, the optimality of mixed bundling extends to moderate-to-weak substitutes and weak complements as well. For strong substitutes, the bundle is so unattractive that mixed bundling converges to pure components with no one buying the bundle. Conversely, for moderate-to-strong complements, the bundle is so attractive that mixed bundling converges to pure bundling (i.e., no one buys just a single component).

Under moderate levels of relative marginal costs (see 4*b*), the cost pressure forces the seller to raise bundle prices, making the bundle less appealing to consumers when products are moderate-to-strong substitutes. Therefore, pure components is more attractive in these cases, and domain of superiority of mixed bundling shrinks compared with the low c/a case. The attractiveness of pure components for moderate complements is interesting. The combination of moderate marginal costs and moderate complementarity pushes bundle prices even higher. Yet the seller cannot ignore the market segments that seek one component only. At optimal levels, the sum of the optimal prices of the components is lower than the bundle price. In other words, mixed bundling collapses to pure components. However, for strong complements, the price premiums that can be gleaned from the bundle are so attractive that the seller is better off withdrawing the components. Pure bundling becomes optimal.

The combination of high relative marginal costs under 4*c* and substitutability makes the bundle even less attractive for substitutes. Thus pure and mixed bundling are less appealing than pure components. Even for independently valued products and weak complements, the high marginal costs keep bundle prices too high. So, pure components is optimal even here. For moderate-to-high complements, the margins from bundle sales are high enough to withdraw the components. Pure bundling becomes optimal.

Clearly, marginal costs and degree of contingency have an interactive effect in determining the optimal strategy. What happens when marginal costs are zero?

COROLLARY 2. Under zero marginal costs, (*a*) mixed bundling is optimal for substitutes, independently valued products, and weak complements; and (*b*) pure bundling is optimal for moderate-to-strong complements.

Pricing Patterns under Mixed Bundling

To assess the relationship between optimal bundle and component prices under mixed bundling, we computed component prices as percentages of the corresponding bundle prices wherever mixed bundling is uniquely optimal. We find that compared with the situation where products are independently valued ($\Theta = 0$), individual prices for complements must be set at a lower percentage of the bundle price. Complementarity encourages the seller to charge higher prices from consumers who seek to buy both products. Yet, component prices must be kept in check to tap consumers who value one item much more than the other. Conversely, component prices for substitutes must be set at a higher percentage of the bundle price.

V. Discussion

In this article, we attempt to delineate the domains of optimality of alternative bundling strategies and to propose optimal pricing patterns for substitutes and complements. In our view, ours is the first study to do so treating complementarity and substitutability as matters of degree and not as a simple dichotomy. Our article revolves around two interrelated questions. Should two complements or substitutes be sold separately, together, or both? How do optimal bundling and pricing strategies for complements and substitutes differ from those for independently valued products?

Relating to the first question, our study suggests that a seller should offer two moderate or strong substitutes separately. Pure components is also appropriate for most complements if the marginal costs are moderate (i.e., not low) relative to the market's maximum willingness to pay. By contrast, the seller should offer two complements purely as a bundle only when marginal costs are relatively low or high (but not moderate). The seller would gain by mixed bundling (i.e., offering the bundle and the individual items) for independently valued products and weak substitutes-complements, provided the variable costs are not too high.

Regarding the second question, Schmalensee (1984) underscores the superiority of mixed bundling for most independently valued products. Our study suggests that pure bundling and pure components have sizable domains of optimality when contingent valuations are relevant. We also find remarkable differences in pricing. Charging higher prices (compared with those for independently valued goods) is optimal not only for complements but also for most substitutes.

VI. Research Limitations and Future Research Directions

To keep our model concise and tractable, we made some restrictive assumptions. These are, arguably, the study's limitations. Even so, they offer opportunities for future research.

For example, our results are based on a bivariate uniform reservation price distribution. Despite its analytical tractability, it limits the scope of our results. We encourage future efforts based on other distributions such as the normal and beta. We could not find closed-form solutions under mixed bundling, a limitation that will also apply to these other distributions.

We assume that reservation prices for the two products across consumers are statistically independent of each other. To be sure, the issue of value dependence that we investigate is an individual-level phenomenon that is different from statistical correlation across consumers. A general model that examines correlation and dependence is likely to be more insightful.

Drawing on parallels in the literature and for parsimony, we assume a constant Θ. As some situations may deviate significantly from this scenario, we made preliminary assessments of the impact of heterogeneity in Θ. Arguing that a minority may perceive a lower magnitude of contingent effects than a majority, we considered two segments: a majority (equaling 80% of the market) perceiving a degree of contingency Θ, and the rest perceiving a degree of contingency of 0.5Θ. We find that the three strategies continue to have unique domains of superiority. However, the domain of optimality of mixed bundling is enlarged. This is because one effect of heterogeneity in Θ is to skew the reservation price distribution of the bundle toward the origin. The power of pure bundling is diminished. This said, more investigation into this area is likely to be valuable. Future efforts could also specify demand by means of a Constant Elasticity of Substitution utility function and evaluate the profitability of each bundling strategy depending on the elasticity of substitution parameter.

Our model assumes a monopolistic framework. Adding the competitive element to the model is likely to provide more valuable insights into the bundling of complements and substitutes.

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