Long-term Forecasting with Innovation Diffusion Models: The Impact of Replacement Purchases

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ABSTRACT
The model presented in this paper integrates two distinct components of the demand for durable goods: adoptions and replacements. The adoption of a new product is modeled as an innovation diffusion process, using price and population as exogenous variables. Adopters are expected to eventually replace their old units of the product, with a probability which depends on the age of the owned unit, and other random factors such as overload, style-changes etc.

It is shown that the integration of adoption and replacement demand components in our model yields quality sales forecasts, not only under conditions where detailed data on replacement sales is available, but also when the forecaster's access is limited to total sales data and educated guesses on certain elements of the replacement process.

KEY WORDS Long term forecasting Diffusion models Durable goods Sales forecasting

Innovation diffusion models have been widely applied in the forecasting of demand for consumer durables, especially in the early years of the product's life. The most popular model in these applications is the one proposed by Bass (1969). This model assumes that the adoption of an innovation depends on a communication process: the new product is first adopted by a few innovators, who in turn generate word-of-mouth, influencing others to imitate. In recent years, several variants of this basic model have been proposed (Dodds, 1973; Sharif and Kabir 1976; Dodson and Muller, 1978; Easingwood, Mahajan and Muller, 1983) in order to obtain a more realistic representation of the adoption process. Nevertheless, as Heeler and Hustad (1980) point out, certain limitations of this modeling approach appear to persist. These authors studied a large number of innovations in several countries and their findings could not match the predictive validity results obtained in previous applications of the model in the U.S.A.

One of the limitations of the model as a forecasting tool stems from the fact that it provides forecasts of adoptions (first time buyers) only, while in general the forecaster's predictive focus centers on the total sales of the product (which include adoptions and replacement purchases). Long term sales forecasting with diffusion models is especially inappropriate, since adoption sales as a proportion of total sales progressively decreases with time. Industry statistics underscore this point emphatically (see Table 1).

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Table 1. Adoption sales*/total sales

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Refrigerators</td>
<td>76%</td>
<td>51%</td>
<td>29%</td>
<td>28%</td>
</tr>
<tr>
<td>Vacuum cleaners</td>
<td>44%</td>
<td>53%</td>
<td>41%</td>
<td>27%</td>
</tr>
<tr>
<td>B &amp; W T.V.</td>
<td>—</td>
<td>63%</td>
<td>25%</td>
<td>30%</td>
</tr>
<tr>
<td>Air conditioners</td>
<td>—</td>
<td>55%</td>
<td>57%</td>
<td>50%</td>
</tr>
<tr>
<td>Mixers</td>
<td>—</td>
<td>77%</td>
<td>72%</td>
<td>45%</td>
</tr>
<tr>
<td>Blenders</td>
<td>—</td>
<td>86%</td>
<td>73%</td>
<td>56%</td>
</tr>
<tr>
<td>Food disposers</td>
<td>—</td>
<td>90%</td>
<td>75%</td>
<td>74%</td>
</tr>
</tbody>
</table>

* Adoption Sales were computed based on yearly changes in the population of homes owning the product (source: Merchandising, March 1960, 1970, 1980).

However, the impact of the forecasting limitation in diffusion models is minimized in the early years of a product's life when adoptions constitute the bulk of total sales; several researchers have used the diffusion model to forecast total sales in this period (Bass 1969; Nevers 1972) although there are potential problems with this approach:

first, model parameters are unstable when only a few data points are available for estimation (Bass 1969; Heeler and Hustad 1980);

second, the application of diffusion models to the total sales of a consumer durable (as commonly found in the literature) carries the implicit assumption that adoptions and replacement purchases are governed by the same communication process between innovators and imitators. It seems reasonable to expect, however, that while potential adopters would be more vulnerable to the word-of-mouth generated by current owners, replacement buyers would depend more on their own personal experience with the product.

Apart from attempting to correct for the above problems, the model proposed and illustrated in the following sections makes a distinction between the two components of demand (adoptions and replacements), treating them as consequences of different behavioural processes. The behaviour of new buyers is modeled through a diffusion process, while the replacement component is obtained through the use of life curves. Despite the distinction between adoptions and replacements, the two components are jointly estimated from a time-series of total sales.

INTEGRATING ADOPTIONS AND REPLACEMENTS IN A FORECASTING MODEL

Although a number of authors have extended diffusion modeling by incorporating repurchases, most use a trial-repeat structure more appropriate for predicting the adoption of non-durable goods. Some of these models (Midgley, 1976; Dodson and Muller, 1978) require consumer panel data for estimation, while others (Lilien, Rao and Kalish, 1981; Mahajan, Wind and Sharma, 1983) account for repurchases through a 'repeat' specification, that is more consistent with the repeat purchase behaviour observed for non-durable goods (where repurchases follow consumption of the good).

With respect to durable goods however, a major motivation for the repurchase is the replacement of an old unit not performing according to the household's requirements. To our knowledge, only two studies have explicitly integrated the replacement demand component with adoption sales to improve long-term forecasting efficiency. Lawrence and Lawton (1981) have
proposed a simple model of replacement purchases in which the service life of the durable is assumed to be constant and known \textit{a priori}; despite these strong assumptions, they present a useful framework. Since their formulation also calls for replacement of every unit at the end of its service life, it provides a basis for decomposing total sales data into adoption and replacement components, thus enabling meaningful long-term estimation of the diffusion model using adoption sales only. However, the assumption that every unit of the product is replaced at the same fixed and known age may mis-specify the actual replacement process.

More recently, Olson and Choi (1985) have presented an elaborate long-term forecasting model which removes the restrictive assumptions found in Lawrence and Lawton (1981). The Olson–Choi model utilizes the Rayleigh distribution to model replacements, and recognizes that the product life is not constant but stochastic.

The aggregate model to be presented here is similar in intent to the Olson–Choi formulation in that it integrates the adoption and replacement demand components to generate long-term forecasts. However, there are several important differences as well, as described later.

Following Olson and Choi (1985), our model assumes that the probability of adoption in the diffusion process depends on the \textit{cumulative number of owners} (homes that have purchased the product at least once), rather than on the \textit{cumulative number of units sold}, as assumed in many previous applications of diffusion models (Bass, 1969; Dodds, 1973; Heeler and Hustad, 1980, among others). Since the proposed model acknowledges that part of the sales are made to replace old units, this distinction is quite important; at any time, the word-of-mouth generated by owners of the product should be a function of the cumulative number of owners, which will tend to be smaller than the cumulative number of units sold.

Our model computes the inventory of units of the product in use each year \((X(t))\) by 'scraping' past purchases using a life-curve approach to be discussed in detail in the next section. Consistent with Lawrence and Lawton (1981) and Olson and Choi (1985), we assume that ownership of multiple units of the product is negligible and that all scrapped units are immediately replaced. The former assumption appears very likely for all the products analyzed in this study (with the possible exception of electric blankets); the latter assumption is plausible since households accustomed to the services of a durable for a few years will perceive it as a necessity, thus triggering immediate replacement after a unit is discarded (Olson and Choi, 1985).

Given the above set of assumptions the inventory of units in use serves as a surrogate for the cumulative number of owners. It is important to note a basic difference between the proposed model and the Olson and Choi (1985) model. In addition to total sales data \((y(t))\), the latter formulation apparently requires data on the cumulative number of adopters \(X(t)\) for the entire estimation period. \(X(t)\) is seldom available for most products and time periods, and may be difficult and costly to estimate reliably on an ongoing basis. Our model requires only total sales data for estimation, which is readily available to forecasters; this parsimony in data input is achieved by computing the population of owners \(X(t)\) within the model as a function of past sales and the replacement process.

Since our model uses a long-term estimation period, it is necessary to explicitly incorporate two key exogenous variables to ensure a realistic representation of the diffusion process. First, the potential number of adopters is assumed to depend on the population of electrified homes at each year. Support for specifying market potential as a function of this predictor is derived from the work of Mahajan and Peterson (1978) who incorporate the effects of population growth (using housing starts as a proxy measure) in the diffusion process for washing machines. Further, the choice of the population of electrified homes appears appropriate since all the products analyzed empirically require electricity for use. We also note that with widespread electrification, this population approaches the total number of U.S. homes in the post-war years.
Second, our model assumes that product price plays an important role in the diffusion process. The need for incorporating price as an exogenous predictor stems from the experience curve literature and related research in the innovation diffusion area (Bass, 1980), which imply that product price and sales growth share a long-term inverse relationship. However, there is no consensus among diffusion researchers on where price should be specified on the diffusion model. Some authors (Robinson and Lakhani, 1975; Bass, 1980; Dolan and Jeuland, 1981) argue that a price decrease could engender an increase in the probability of adoption. Hence a price reduction would only motivate potential buyers to make an earlier adoption, with no impact on the total market for the product. On the contrary, other researchers (Chow, 1967; Mahajan and Peterson, 1978; Horsky and Simon, 1983) assert equally convincingly that a lower price would place the product within the budget limitations of a greater number of buyers, thus affecting the market potential for the product. Under this view, price changes would have a long-term impact, by increasing/decreasing the total pool of potential buyers who would eventually buy the product.

Clear empirical evidence on the relative superiority of these two alternative approaches to specifying price in the diffusion equation is lacking in the literature (see Mahajan and Muller, 1979 for a good discussion). Given this research void, we follow the approach of Mahajan and Peterson (1978) who recommend specifying market potential as a function of all marketing mix variables, besides the population of households.

In developing the proposed model, we use the term ‘survivors’ to connote product units that are still in use since they continue to perform according to the owner’s expectations; ‘dead’ products, on the other hand, represent units that have failed to meet these expectations.

Let,

\[ y(t) = \text{Sales of the product at year } t \]
\[ \Pr(t) = \text{Price Index (in 1967 dollars) such that } \Pr(1) = 1 \]
\[ \text{Pop}(t) = \text{Population of Electrified Homes} \]
\[ a, b, \alpha \text{ and } \beta = \text{parameters to be estimated} \]

Then, the proposed model can be expressed by,

\[ y(t) = [a + bX(t)][\alpha \text{ Pop}(t)\Pr^\beta(t) - X(t)] + r(t) + e(t) \] (1)

where,

\[ X(t) = \sum_{i=1}^{t} M(i) \cdot y(t - i) \] is the total number of units in use at the beginning of year \( t \), assuming that all ‘dead’ units are replaced immediately. Note that this represents the cumulative number of owners.

\( M(i) \) is \% of units from a lot which are expected to ‘survive’ until \( i\)-years after purchase. Although this survival function \( M(i) \) is continuous over time, it is used here on a discrete form because sales are measured at discrete time intervals.

\[ r(t) = \sum_{i=1}^{t} [M(i - 1) - M(i)] \cdot y(t - i) \] is the number of units that have ‘died’ or need replacement at year \( t \). Note that \([M(i - 1) - M(i)]\) represents the \% of units produced \( i \) years ago that have ‘died’ at year \( t \).

\( e(t) \) is random disturbances.

The parameters \( a \) and \( b \) represent the innovation and imitation coefficients respectively; note that our imitation coefficient is not divided by the market potential as in Bass (1969) and other diffusion studies. Therefore these estimates are not directly comparable with prior research. The parameter \( \alpha \) represents the ultimate penetration (proportion of the total population of homes...
expected to adopt the product), if price was kept at its original level. Parameter $\beta$ measures the impact of price changes on the ultimate penetration achieved by the product.

**MODELING THE REPLACEMENT OF DURABLE GOODS**

Before starting the discussion on the replacement component of demand, some terms need to be defined. **Replacement Rate** ($m(t)$) refers to the probability that a unit is replaced at age $t$, given that it had survived until then. **Survival Function** ($M(t)$) represents the probability that a unit will remain in use until age $t$, or, in the aggregate case, the proportion of a lot that will survive until age $t$. The **Replacement Distribution Function** ($f(t)$) provides the unconditional probability that a unit is replaced at age $t$, or, in the aggregate case, the proportion of a lot expected to be replaced at age $t$.

The following relationships hold among these probabilities:

$$M(t) = 1 - \int_0^t f(i) \, di$$  \hspace{1cm} (2)

$$f(t) = m(t) \ast M(t)$$  \hspace{1cm} (3)

The choice of an appropriate probability density function (pdf) for $f(t)$ is crucial in modeling the replacement component accurately. We discuss below the relative merits and demerits of several pdf’s to justify our choice of pdf for $f(t)$.

One of the simplest Replacement Distribution Functions is the exponential distribution ($f(t) = e^{-\lambda t}$). Despite its simplicity (the inventory in use at each year is depreciated at a constant rate $\lambda$), this replacement distribution implies that the replacement rate is independent of age.

However, it is well accepted among researchers in the field of product reliability, that the replacement rate should relate to age, specially when the replacement or failure depends on aging and wear (Chhikara and Folks, 1977). The distribution of first failures in reliability studies (or of replacements, in our case) is commonly represented by a lognormal distribution, which implies a non-monotonic failure (or replacement) rate; the failure (replacement) rate is assumed to initially increase starting from zero (at age 0), and to eventually decrease towards a zero asymptote (Watson and Wells, 1961). In other words, this distribution assumes that if a unit survives up to an old age, its probability of failing at that age would be null.

In order to avoid this unrealistic assumption, Chhikara and Folks (1977) proposed the use of an Inverse Gaussian distribution of failures, which allows for a non-zero failure rate at old ages. Nevertheless, this distribution of failures still assumes that the failure rate is null or minimal at very early ages. However, while dominated by age, the replacement rate of a durable good is also affected by a multitude of ‘random’ factors unrelated to product’s age, such as overload (underload)—very heavy (light) use of a product which advances (delays) the replacement probability determined by its ‘age’—and style changes (e.g. early replacement of a still performing unit with another new unit whose ‘style’ differences better accommodates the consumer’s preferences), etc. Hence, one could expect the replacement rate to be significant, even in the early life of the unit. In fact, replacement rates for consumer durables tend to increase monotonically with age (and not necessarily linearly), starting from a non-zero value. This can be seen in Figure 1, which plots actual replacement rates for four consumer durables, observed in a 1972 USDA survey (see Ruffin and Tippett, 1975).

To summarize, the choice of the pdf of $f(t)$ must allow for the following empirically observed characteristics in $m(t)$ seen in Figure 1:

(a) $m(t)$ must start from a non-zero value

(b) $m(t)$ must increase monotonically (not necessarily linearly) with time
It is interesting to note that the Rayleigh \( f(t) \) distribution chosen by Olson & Choi (1985) does not capture either of the empirical \( m(t) \) characteristics outlined above. This pdf provides only for a \( m(t) \) that starts with a zero value and increases linearly over time (see Appendix A, part 1 for details). Further, these authors clearly state that the Rayleigh pdf captures only age-related replacements, assuming that the effect of random factors on replacement sales is unimportant.

Graphically translated, the two characteristics of \( m(t) \) discussed above lead to skewed and truncated replacement distributions \( f(t) \) as shown in Figure 2. An adequate and convenient formulation for this type of replacement distribution was proposed by Gumbel (1933) and Kimball (1947). This system of survival functions (hereafter referred to as the \( h \)-type functions) uses two parameters to characterize the probability of survival up to a given age: the average service life (L), which represents the expected life of a unit, and the shape parameter (h), which determines the particular type of replacement rate implied by the survival function. Mathematical details of this system of survival functions are presented in Appendix B.

The replacement distributions and replacement rates associated with the \( h \)-type survival functions for an average life of \( L = 10 \) and different values of the shape parameter \( h \) are displayed in Figure 3. One can notice from this figure that this system of functions offers a wide-ranging flexibility in life-curve specification: it accommodates the constant rate distribution (when \( h \) tends to \( -\infty \)), and can also allow for increasing, non-linear replacement rates, at different growth rates, depending on the shape parameter. In contrast, the Rayleigh pdf lacks such flexibility (see Appendix A, part 2).

The \( h \)-type family also allows for non-zero replacement rates at the very early ages, which is commonly observed in the replacement analysis of consumer durables. Further, Kimball (1947, p. 357–358) demonstrates that the replacement distribution associated with this family of survival functions can be decomposed into one part which represents replacements due to ageing, and another due to random effects independent of age (which are likely to be observed for consumer durables).

The characteristics discussed above make the \( h \)-type family of functions quite appropriate for representing the replacement process of consumer durables. In fact, the comparison of observed
Fig. 2. Distribution of replacements for some consumer durables
Fig. 3. Replacement distribution and replacement rates: h-type family
replacement frequencies (from the 1972 USDA survey) and the replacement distribution functions obtained from the \(h\)-type family (see Figure 2), shows a reasonable fit, considering the error associated with the observed frequencies; these frequencies are based on cross-sectional replacement data, rather than on the longitudinal analysis of a cohort. Figure 2 also shows that the \(h\)-type replacement distribution is more flexible than the Rayleigh distribution, best accommodating the various shapes observed in the life curves of consumer durables. The \(h\)-type and Rayleigh replacement distribution functions displayed in Figure 2 were obtained through the maximum-likelihood estimation (MLE) of \(h\) and \(L\), based on the observed replacement frequencies.

Therefore, the \(h\)-type family provides the means for forecasting the inventory of units in use and the replacement sales described in Equation (1). If data on the replacement rate for the product under study are available, the parameters \(h\) and \(L\) can be directly estimated, and the problem of estimating and forecasting the replacement component of the proposed model would be solved by substituting Equation (B1) (Appendix B) for \(M(t)\) in our proposed model (Equation 1). Otherwise, the parameters of the Survival Function \((h\) and \(L\)) would have to be estimated simultaneously with the diffusion parameters \((p, q, \alpha \) and \(\beta\)). These possibilities will be discussed next, along with empirical applications of the model.

**EMPIRICAL ILLUSTRATIONS OF THE PROPOSED FORECASTING MODEL**

Two situations will be considered in these empirical applications. In the first, it will be assumed that the forecaster has access to data regarding the replacement distribution for the product under study, perhaps from a cross-sectional survey among owners or, ideally, from the longitudinal monitoring of a cohort of purchased units of the product. In the second situation, it will be assumed that the forecaster can only make some educated guess about the nature of the replacement process, i.e., a rough estimate of the ratio between the maximum life and the average service life of the product.

Annual manufacturer shipments (in million units) and average prices for major household appliances, and the number of electrified households are published by *Merchandising* (special issues). Since price indices (U.S. Bureau of Labor Statistics) were available only for some appliances and for the most recent years, the average price (at constant dollars) was used as a proxy, whenever necessary. Based on this information, a price index (at constant dollars) was constructed to each product, such that \(PR(1) = 1\).

**First condition: replacement data are available**

Replacement data for several consumer durables can be found in studies to estimate the service life of several household appliances (Jaeger and Pennock, 1957; Pennock and Jaeger, 1964; Ruffin and Tippett, 1975). The data used in this application is available from actuarial tables based on a 1972 cross-sectional survey of 11,696 households conducted by the U.S. Department of Agriculture. Details on the construction of these tables can be found in Ruffin and Tippett (1975).

Let \(g(t)\) denote the observed proportion of units replaced at age \(t\). Then, the average service life can be directly estimated as the average age of replacement \(L = \int_{0}^{\infty} i \cdot g(i) \, di\). Estimates of the shape-parameter \(h\) can then be obtained graphically, using an abacus published by Kimball (1947, table 2 and figure 5). Maximum Likelihood estimates of both parameters can be easily obtained by finding the values of \(h\) and \(L\) such that,

\[
\max_{(h, L)} LL(h, L) = \int_{0}^{\infty} g(i) \cdot \log[f(i, h, L)] \, di
\] (4)
Table 2. Parameter estimates for the $h$-type family based on the 1972 USDA survey data

<table>
<thead>
<tr>
<th>Product</th>
<th>Shape parameter $h$</th>
<th>Average service life $L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B &amp; W T.V.</td>
<td>1.9</td>
<td>10.9</td>
</tr>
<tr>
<td>Color T.V.</td>
<td>2.7</td>
<td>10.3</td>
</tr>
<tr>
<td>Dishwashers</td>
<td>1.8</td>
<td>9.4</td>
</tr>
<tr>
<td>Dryers</td>
<td>2.4</td>
<td>13.4</td>
</tr>
<tr>
<td>Electric ranges</td>
<td>1.4</td>
<td>10.7</td>
</tr>
<tr>
<td>Freezers</td>
<td>1.7</td>
<td>17.9</td>
</tr>
<tr>
<td>Gas ranges</td>
<td>0.73</td>
<td>12.5</td>
</tr>
<tr>
<td>Refrigerators</td>
<td>1.8</td>
<td>14.3</td>
</tr>
<tr>
<td>Washers</td>
<td>2.3</td>
<td>11.2</td>
</tr>
</tbody>
</table>

As can be easily seen from Equation (B2) (Appendix B), $f(.)$ in Equation (4) is a function not only of time, but parameters $h$ and $L$ as well. Table 2 lists the MLE estimates of $h$ and $L$ parameters for several household appliances, obtained by using the above procedure on the 1972 USDA survey data.

Once these parameters are known, the survival probabilities at any age can be computed, and hence, the replacement part of the forecasting model ($\lambda(t)$ and $r(t)$) can be directly obtained using Equation (1). The estimation problem then is reduced to a typical innovation diffusion model, which can be estimated through non-linear least squares. The estimation algorithm developed for this purpose uses past sales and the survival function to compute estimates of the inventory of units in use and replacement purchases at each year, and then uses these results as inputs in the estimation of the diffusion model. Results from this procedure for Refrigerators are presented in Table 3 and Figure 4, in comparison to the results of the Olson–Choi and Diffusion-only models, applied to total sales. This latter model is similar to the proposed model, except that it considers all sales as adoptions, ignoring the replacement component of demand.

For this comparison, the first 35 years of the product’s life were used in the estimation of the models*. The survival function $M(i, h, L)$ was assumed to be known a priori, using the $h$-type family and the Rayleigh distribution, with the shape-parameter (for the $h$-type family) and average service life listed in Table 2. The parameter estimates obtained were then used to forecast sales for the later 24 periods.

It is evident from Figure 4 that the forecasting performance of the proposed model (MAPE = 12%) is superior to the diffusion-only (MAPE = 31%) and Olson–Choi (MAPE = 20%) models. The flexibility of the logistic formulation used by the diffusion-only model allows it to fit quite reasonably to the estimation data (see Figure 4). However, the exclusion of the replacement component of demand resulted in the under-forecasting of future sales, and also produced some counter-intuitive parameter estimates. For instance, the diffusion-only model predicts that the potential number of adopters would eventually be 40% higher than the number of homes in the country, even if prices were held at their original levels. Although some of these units might be explained by the ownership of multiple units, it seems reasonable to expect that this discrepancy would be mostly due to the replacement component of demand not considered by the diffusion model.

* For all analyses pertaining to Refrigerators, Toasters and Vacuum Cleaners in this paper, data from the first 35 years of the product’s life were used. Since this data period included the World War II years (during which there was no significant production/sales) for each of the three products, we incorporated a war dummy variable for this period in the regression equation.
Table 3. Parameter estimates: first condition refrigerators

<table>
<thead>
<tr>
<th></th>
<th>$h^*$</th>
<th>$L^*$</th>
<th>$a$</th>
<th>$b$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>Dummy</th>
<th>$R^2$ (FIT)</th>
<th>MAPE (FOR.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed model</td>
<td>1.8</td>
<td>14.3</td>
<td>0.190</td>
<td>1.01†</td>
<td>0.904†</td>
<td>-0.190</td>
<td>-2.701†</td>
<td>0.89</td>
<td>12%</td>
</tr>
<tr>
<td>Diffusion-only</td>
<td>—</td>
<td>—</td>
<td>0.229†</td>
<td>0.390†</td>
<td>1.40†</td>
<td>-0.251</td>
<td>-2.470†</td>
<td>0.86</td>
<td>31%</td>
</tr>
<tr>
<td>Olson-Choi</td>
<td>—</td>
<td>14.3</td>
<td>0.104</td>
<td>0.848†</td>
<td>$193 \times 10^6†$</td>
<td>-3.100†</td>
<td>0.88</td>
<td>20%</td>
<td></td>
</tr>
</tbody>
</table>

* Parameters fixed to the values listed in Table 1.
† Statistically significant at the 0.05 level.

Second condition: replacement data not available

Under this second condition, it would be desirable to estimate both parameters of the survival function, along with the diffusion parameters, based on total sales data. This joint estimation, however, is not feasible, unless there are some additional data regarding the replacement component of sales (e.g., either direct data on historical replacement purchases, or data on the inventory of units in use). Therefore, either one of the survival function parameters ($h$ or $L$) has to be obtained exogenously. Our experience with the model indicated that its results are somewhat sensitive to mis-specifications of the average service life ($L$). Furthermore, some guidelines are available to make educated guesses regarding the shape parameter ($h$), as presented below.

Let $S$ be the age such that $M(S) = 0.0001$. This age can be viewed as a proxy for the maximum service life for the product under study. Oates and Spencer (1962) demonstrate that the ratio between this maximum life and the average service life ($S/L$) is directly related to the shape parameter $h$. In fact, these authors have tabulated this relationship, such that the forecaster can...
<table>
<thead>
<tr>
<th>Product</th>
<th>Sample Period</th>
<th>$h^*$</th>
<th>$L$</th>
<th>Proposed model</th>
<th>$a$</th>
<th>$b$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>Dummy†</th>
<th>$R^2$ (FIT)</th>
<th>MAPE (FOR.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Refrigerators</td>
<td>1925–59</td>
<td>1.75</td>
<td>15.8</td>
<td>$a \times 10^{-1}$</td>
<td>0.190</td>
<td>0.917†</td>
<td>0.960‡</td>
<td>-0.176</td>
<td>-2.670‡</td>
<td>0.89</td>
<td>11%</td>
</tr>
<tr>
<td>Vacuum cleaners</td>
<td>1922–56</td>
<td>1.75</td>
<td>16.1</td>
<td>$b \times 10^{-5}$</td>
<td>0.429†</td>
<td>0.624‡</td>
<td>0.978†</td>
<td>-0.514</td>
<td>-1.800‡</td>
<td>0.87</td>
<td>18%</td>
</tr>
<tr>
<td>Toasters</td>
<td>1922–56</td>
<td>1.75</td>
<td>13.3</td>
<td></td>
<td>0.181†</td>
<td>1.14†</td>
<td>0.965‡</td>
<td>-0.00</td>
<td>-2.780‡</td>
<td>0.92</td>
<td>7%</td>
</tr>
<tr>
<td>Electric blankets</td>
<td>1948–72</td>
<td>1.75</td>
<td>15.5</td>
<td></td>
<td>0.084†</td>
<td>0.695‡</td>
<td>0.814‡</td>
<td>-0.057</td>
<td>—</td>
<td>0.95</td>
<td>14%</td>
</tr>
</tbody>
</table>

* The $h$-parameter was fixed to 1.75 based on the assumption that these products have a maximum life approximately equal to three times their average service life.
† This dummy variable was used to account for zero sales during the World War II years.
‡ Statistically significant at the 0.05 level (standard error of estimate not available for $L$).
obtain an estimate of $h$ based on an assumed $S/L$ ratio (Oates and Spencer 1962, table 2, p. 454). An extract of this table is shown below.

<table>
<thead>
<tr>
<th>Ratio $S/L$</th>
<th>1.5</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>4.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h$-parameter</td>
<td>7.50</td>
<td>3.75</td>
<td>1.75</td>
<td>0.76</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Therefore, whenever the forecaster can make an estimate of the maximum life span for the product, expressed in terms of average service life units, s/he will be able to make an educated guess of the shape parameter. Our experience with the $h$-type survival function (see Table 2) indicates that this parameter often falls within a narrow range ($0.5 < h < 2.5$).

The algorithm developed to estimate the proposed model under this second condition seeks the parameter set (which includes the diffusion parameters and the average service life $L$) that minimizes the sum of squared residuals ($e(t)$). These residuals are computed by first using the $h$-type survival function (defined by the pre-specified $h$ parameter and the estimate of the average service life $L$) to transform past sales into the inventory of units ($X(t)$) and replacement sales ($r(t)$), and then applying these values into Equation (1). Least-squares estimates of the parameters ($a, b, x, \beta, L$) are obtained using a Gauss-Newton non-linear optimization algorithm (Fletcher, 1972).

Four empirical applications of the proposed model, under this second condition, are reported next. In all cases (Refrigerators, Vacuum cleaners, Toasters and Electric blankets), it was assumed that a unit would last for, at most, three average service lives. According to the table listed by Oates and Spencer (1962), the respective shape-parameter is $h = 1.75$. The results of the analyses are reported in Table 4 and Figure 5. In each case, the models were estimated based on an estimation period (35 years for Refrigerators, Vacuum cleaners and Toasters, and 25 years for Electric blankets), and then used to forecast total sales for the remaining years.

Unfortunately, a direct comparison of our proposed model with the one by Olson and Choi (1985) under this second condition (no replacement data available) was not possible, since (to our best knowledge) this model uses the population of owners, unknown under this second condition, as a predictor. We attempted to use the number of surviving units (as per our proposed model) as an endogenous 'proxy' for this predictor, but this variant of the Olson–Choi model had failed to converge for all the four products tested, tending to a solution with infinite average service life.

Although the goodness-of-fit obtained with the proposed model was not substantially different from the simple diffusion model during the estimation period (when the replacement component of sales was not as important), its forecasting performance was clearly superior. The acknowledgement of an important component of demand seems to make a substantial difference in the ability to forecast long-term sales. Furthermore, the parameter estimates obtained with the proposed model have a more reasonable interpretation. For example, the diffusion-only model prescribes an eventual saturation (at minimum) of 1.40, 1.50 and 1.60 for Refrigerators, Vacuum cleaners and Toasters, respectively. In other words, since it ignores an important demand component, this model predicts that the eventual number of adopters for these products is more than 40% larger than the number of electrified homes in the U.S. On the other hand, the proposed model predicts that 96% of the electrified homes in the U.S. would eventually have a refrigerator (98% for vacuum cleaners and 96% for Toasters), if prices were held at their initial level. Since there was a substantial reduction in prices since then, the saturation level may reach higher levels (possibly above 100%, due to the ownership of multiple units). The impact of the price reduction on the saturation level achieved by the product is measured by the $\beta$ parameter; we note that this parameter always has the expected negative sign although, contrary to our expectations, it is generally insignificant.

A comparison of the estimated average service life for refrigerators ($L = 15.8$) with the estimate obtained from the USDA survey (Table 2) shows a discrepancy of 1 year. However, one should remember that the latter was obtained from cross-sectional data, collected in 1972, while the
Fig. 5. Comparison of fit and forecasts.
Fig. 6. Sensitivity analysis
estimate obtained with the proposed model is the one which best fits to the sales figures during the whole estimation period.

It is worth mentioning that the actual population of homes and price index were used in computing the forecasts displayed in Figures 4 and 5. Obviously, in a real forecasting situation this information would not be readily available. Reasonable estimates of the number of homes may be obtained, however, from the expected population of households. Projections of prices for consumer durables can also be obtained, by estimating experience curves based on past price and production data (Yelle, 1979), or by incorporating the experience curves into the diffusion model (see Bass, 1980; Dolan and Jeuland, 1981).

Furthermore, the results displayed in Figure 5 might be dependent on the right choice of the shape-parameter $h$. In order to verify the sensitivity of the forecasts to the particular value used for $h$, the model was also estimated with two different 'guesses' ($h = 0.5$ and $h = 2.5$). A comparison of the results obtained (see Figure 6) showed only minimal differences in the forecasts, indicating that the model is reasonably robust to mis-specifications of the shape-parameter, within a reasonable range of values.

**SUMMARY, LIMITATIONS AND CONCLUSIONS**

The diffusion literature has long neglected the issue of replacement purchases. This demand component will 'contaminate' forecasts when total sales data are used to generate long term sales predictions based on the diffusion model.

Recent research, however, has sought to address this important issue (Lawrence and Lawton, 1981; Olson and Choi, 1985). Our presentation here follows the spirit of these research efforts, but offers a more generalized model specification with the following unique features:

(a) Our model assumes that replacement sales occur whenever an existing unit of the product fails to perform according to the owner’s expectations (this subsumes replacements both due to age-driven and random factors). This is accomplished by specifying a theoretical replacement function ($h$-type family) that accommodates certain empirically observed characteristics such as nonlinearity as well as a non-zero replacement rate in the early years.

(b) Unlike other models that require data on the number of owners every time period (which may be unavailable, or expensive to collect), our model only requires the knowledge of total sales data. This parsimony in data input is achieved by enabling the forecaster to select the value of the life-curve shape parameter via educated guesses of the $S/L$ ratio; this permits endogenous estimation of replacements each time period.

(c) The $h$-family of life curves provides a wide ranging flexibility/choice of life curve specifications that is unmatched by other pdf’s such as the Rayleigh, Lognormal and Inverse-Gaussian.

It is also important to highlight the limitations of the model. First, our model assumes that the survival rate $M(i)$ is independent of the time of adoption. For instance, the model assumes that if refrigerators were adopted by some households in 1925, the percentage of survivors in 1926 is the same as the percentage of survivors between 1978 and 1979. One can question this assumption based on product improvements stemming from technological advances during the estimation period used. Our model attempts, however, to find the life curve which best fits to the entire estimation period, however long this may be.
Second, the proposed model assumes that the replacement of all durables is a technological necessity, due to deterioration with age. The impact of other factors unrelated to the age of the owned unit was only considered as a random phenomenon which raises the probability of replacement at the early ages. Although this simplifying assumption may be acceptable for forecasting purposes (perhaps desirable, since it makes the model more parsimonious and hence more robust), it might provide only a partial understanding of this important component of demand. One can argue that the replacement phenomenon is not only constrained by the performance of the units in use, but is, in fact, an economic decision involving the trade-offs among the costs/benefits of repair and replacement. Studies regarding this decision can be found in the economics literature (Su, 1973; Schmalensee, 1974). However, while emphasizing the economic aspects of this process, these studies tend to make simplistic assumptions regarding the physical decay, usually specifying a fixed service life, or a constant rate of decay for the existing units of the product.

Another simplifying assumption implied by the model is that all repurchases are made for the purpose of replacing old units, thus ignoring the repeat-purchase component of demand. Repeat purchases may occur whenever the household desires to own more than one unit of the product, (e.g. television sets, radios, refrigerators, etc.). The effect of ignoring this component, when needed, may be an overestimation of the market saturation (z). Future research must address all three demand components in long-term sales forecasting of durables: adoptions, replacements and repeat sales.

REFERENCES

APPENDIX A

Part 1

The Rayleigh pdf for the Replacement Distribution is

\[ f(t) = 2\delta t e^{-\delta t^2} \]  (A1)

The Survival Function related to this pdf is

\[ M(t) = \int_t^\infty 2\delta e^{-\delta i^2} di \]

\[ = e^{-\delta t^2} \]  (A2)

The replacement rate is

\[ m(t) = f(t)/M(t) \]

\[ = 2\delta t \] (which is linear with age \( t \) and has a zero origin).  (A3)

Part 2

As demonstrated by Olsen and Choi (1985), the mean and variance for the Rayleigh pdf are:

\[ L = 1/2 \sqrt{\pi/\delta} \]  (A4)

\[ \sigma^2 = (1 - \pi/4)/\delta. \]  (A5)

The two equations above indicate that the mean and variance are directly related via the common parameter \( \delta \); once this parameter is fitted to the average service life (\( L \)) of the product, the variance
of the pdf (and hence, the shape of the life curve) will be automatically fixed. While this fact leads to a parsimonious specification of the life curve, it will also constrain its shape.

APPENDIX B

Let \( h \) denote a shape parameter and \( L \) represent the average service life for the product under study. Let us also define,

\[
w = h + \phi(-h)/\Phi(-h),
\]

where \( \phi(.) \) denotes the standard normal pdf, and,

\[
\Phi(x) = \int_{-\infty}^{x} \phi(z) \, dz.
\]

Then, Kimball (1947) demonstrates that the following function represents a family of survival functions with average service life \( L \) and with a replacement rate determined by the value of the shape parameter \( h \):

\[
M(t) = \frac{\Phi(wt/L - h)}{\Phi(-h)}
\]

The replacement distribution function associated with the survival function above is a truncated normal,

\[
f(t) = \frac{w\phi(wt/L - h)}{L\Phi(-h)},
\]

The replacement rate associated with the functions above is, in general, monotonically increasing, but can also be constant when \( h \) tends to \(-\infty\). The replacement rate is computed from these functions as,

\[
m(t) = \frac{w\phi(wt/L - h)}{L\Phi(wt/L - h)}
\]

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